Brought-Forth Possibilities for Attentiveness in the Mathematics Classroom

A Response to Elizabeth Mowat & Brent Davis

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Embodied mathematics presents a significant, and welcomed, shift away from the traditionalist, objectivist mathematics education that continues to dominate schooling. Elizabeth Mowat and Brent Davis' *Interpreting Embodied Mathematics using Network Theory: Implications for Mathematics Education* is a fine example of embodied mathematics. It represents a movement toward a conception of mathematics (and knowledge in general) as a somatically-rooted, tacitly-felt and emotionally-experienced activity. We understand mathematics as we do because we are radially-symmetrical, gendered and sexual, sensory-motor sensitive, and conscious beings, who are, evolutionarily, socio-culturally, and historically, situated. Biology and knowing co-emerge, to use Francisco Varela's understanding, in the very process of living (Varela, 1979; Varela, Thompson and Rosch, 1991). That knowing is irreducibly grounded in biology suggests that knowing needs to be re-conceptualized as embodied action.

Articulating mathematics as an embodied, complex unity, Elizabeth Mowat and Brent Davis theorize how mathematical understanding might be generated, both individually and collectively, and what teachers might do to facilitate this process. The authors build on Lakoff and Núñez's (2000) theory of embodied mathematics, which conceives of knowledge as emergent through connections among conceptual metaphors and sensori-motor experience. Network theory analysis, suggest the authors, allows for a deep understanding of the underlying metaphorical, structural dynamics that bring

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forth and transform mathematical cognition. Equally important, network theory offers embodied explanations for the breakdowns (or cascading failures) of mathematical understanding. Envisaging mathematics as a scale-free structure, one can see why attempts to bolster major source domains used in mathematics metaphors tend to fail because, as the authors put it, "Hubs are still hubs" (p. 22). Instead, Mowat and Davis suggest a different pedagogical approach. They suggest that the introduction of *weak* links might actually strengthen networks of metaphor interconnectivity.

I understand weak links to represent non-essential, and hence dispensable, "bridges" or "shortcuts" that provisionally connect elements of a network. Locally-connected, these weak links can generate global understandings, but if compromised, do not lead to the cascading failures typical of more centralized networks. The generation of variously-connected weak links of metaphors might promote in mathematics learners the propensity to understand mathematics in multiple ways, and hence to develop an awareness that learning is an exercise in multiplicity. I return later to this notion of multiplicity and ways it might promote weakly-linked metaphorical interconnectivity.

I see in Mowat and Davis' research problematic a provocation to explore different possibilities: (1) Understanding mathematics as a complex unity emerging through the intensifications or dissipations of various metaphorical connections; (2) Using network theory to understand the metaphorical structure of mathematics; and (3) Seeing teachers as having a grounding role in helping children to generate more distributed—and weakly-linked—networks of understanding. It is the third possibility that particularly compels me. What might an understanding of network theory do for the day-to-day interaction of teachers and children? What might teachers themselves *do* to be able to support children in increasing "the number of connections among conceptual domains [that] would have the desired effect of reducing the network's dependence on its hubs" (p. 23)?

Generating Neuronal Connections Among Conceptual Domains

The activities involved in the generation of neuronal links of conceptual metaphors, weak or otherwise, are, by nature, not visible to the perceiver's direct awareness. They occur at a neuronal domain of interaction, and are characterized by what Varela (1992) called multidirectional multiplicity. Communication among neuronal ensembles is triggered by internal and environmental perturbations that one is predisposed to be sensitive towards. Such communication continues to resonate until a coherent perception—an act of distinction—is brought forth from within one's entire phylogenically and ontogenically emergent nervous network. This brought-forth act of distinction is, in turn, the perception(s) that one is predisposed to understand at any given moment in time. Put differently, locally-operating ensembles of neurons stimulate one another to alternatively compete and cooperate, and in doing so, produce global understandings. Perception works such that in the time that one is able to perceive a coherent perception—that is, by the time one *knows*—one's nervous system has already constituted a brought-forth world and coterminous sense of self-hood associated with it

(Maturana and Varela, 1980, 1998). We cannot step outside ourselves, so to speak, to observe the moments of potential bifurcation that generate our current realities.

Implications include that it is not important or even necessary to know beforehand which neuronal ensembles will connect with which others. What is necessary, however, is that one participates in qualitatively rich, diverse, and emotionally-felt experience (understood broadly as encompassing both sensori-motor forms and more conceptual forms), each capable, using the language of chemistry, of reacting or being connected with some other experience(s). Given these conditions of closure, at a certain critical threshold, what Stuart Kauffman (2000) calls an autocatalytic network would spontaneously emerge. Kauffman relates, in his "button" analogy, how such an autocatalytic network emerges:

> Consider 10,000 buttons on a hardwood floor and a spool of red thread. You pick a random pair of buttons, break off a length of red thread, and tie two buttons together. Now just repeat the process picking successive random pairs of buttons, including ones that are already paired with another button, and tying them together with pieces of red thread. Every now and then, pause to lift a randomly chosen button off the floor and check how many other buttons you lift up with it as a single connected cluster of buttons. (2000, p. 35)

At the beginning, every time a button is randomly lifted, the chances are that it is not connected to a red thread or to other buttons, or that the button is connected, but only to a small cluster of thread and buttons. At some intermediate time, a larger cluster of buttons and thread begins to form. After passing a certain threshold—a critical threshold of diversity—the addition of only a few more threads will greatly enhance the connectivity of the button-thread network to produce a giant cluster. This threshold is known as a phase transition, a mathematical threshold across which the addition of some input x needs to be escalated only marginally in order to produce overwhelmingly relational results. At this threshold of phase transition, there is "a sudden jump to a richly connected network" (Kauffman, 2000, p. 37) that has spontaneously formed. Given these dynamic conditions, iterative feedback in this global order recursively produces "coherent, self-reinforcing web[s] of reactions" (Waldrop, 1992, p. 123). No external force or work is needed to produce this order. It emerges spontaneously—what Kauffman refers to as "order for free" (1993)—as an emergent property of the internal relationality among the different locally-connected nodes and hubs constituting a neuronal network.

This buttons and thread analogy (known formally as a random graph) is of course simplistic, and does not adequately demonstrate the dynamically rich nature of cognition. Neither does it make the necessary distinctions between centrally-hubbed networks and more weakly-linked ones. It shows though that prescriptive attempts at engineering networks of metaphor relationality do not work, nor is it important to include particular kinds of "buttons" or "threads" in the reaction mix, in order to produce a network of understandings, which emerges, at *certain conditions*, "for free."

If mathematics understanding can emerge for free in conditions of autocatalytic closure, what can teachers do to contribute to these necessary conditions? And if, as

Mowat and Davis invite us to consider, weak links promote more robust networks of connectivity because multiple metaphorical connections generating an understanding are potentially stronger than one or a few connections, what can teachers do to support children to develop such multiplicity?

Mowat and Davis suggest that when teachers regularly introduce novel connections across domains, they can help learners generate more robust networks of concepts (p. 23). The cultivation of this propensity, as the authors put it, requires ongoing professional development to help teachers develop: (1) an understanding of the importance of conceptual metaphors for cognition; (2) different strategies for introducing metaphors; and (3) increased knowledge of the different metaphors that connect mathematics concepts (p. 26). These conditions are indeed essential, as they provide the knowledge base for introducing diverse metaphors in the complex classroom. I suggest that an additional, and I think indispensible, condition is attentiveness.

Moment-to-Moment Attentiveness as Experience and Weak Links

Attentiveness is the cultivated capacity to be responsive *in* the present moment of experience. Promoting attentiveness in classrooms is crucial to supporting children in their efforts to make embodied connections, both strongly- and weakly-linked, among different metaphors. It allows teachers to develop a responsive awareness of the unique ways learners are making connections and of the appropriate timing and context to introduce them to new metaphors that might encourage the generation of weak links. Attentiveness in this sense is to be understood as a listening responsiveness, or as what Humberto Maturana has referred to as "listening to the listening" (in Maturana and Poerksen, 2004, p. 131-32), the development of which, requires embracing a non-judgmental responsiveness as mutually reciprocal openness, and an empathy and concern for others. Judgment and analysis have their place of course, but they need to come after genuine relationality has taught us something new and has provided us with a deeper insight into the multiplicity that emerges when we experience different, and often far-removed, ways of understanding a particular phenomenon. The practice of attentiveness then, is the practice of moment-to-moment awareness of the nature of one's habituated on-going relational experience, so it can momentarily come to a stop.

The suspension of judgment required is more than simply the avoidance of "jumping to conclusions", to use a common colloquialism. Rather, it demands attending to certainties as they play themselves out in the moment—the commitments, truths, and beliefs taken up as fact, which typically obscure attentiveness to perceive anew. It involves experiencing how one's beliefs, habits, and judgments function forcefully in limiting the multiplicity of ways in which one generates understanding. The promise of attentive awareness is that heedful listening accompanied by a suspension of judgment might open up embodied awareness that the world one experiences is not *the* world but *a* world which we bring forth with others. "The universe," as Maturana (in Maturana and Poerksen, 2004, p. 39) remarks, "changes into a multiverse within which numerous realities are valid by reference to different criteria of validity."

One is attentive in experiencing "what one's mind is doing as it does it" (Varela, Thompson, Rosch, 1991, p. 23). One cannot will oneself to be attentive—that would be concentration—but allow one's attentiveness to unfold, in what Krishnamurti (1954, 1997) referred to as "effortless effort." Attentiveness is thus not about raising self-esteem and feeling good about oneself, but about being relationally responsive with others. Understood in this relational way, attentiveness becomes not a reflection *on* experience—and thus there is no answer to the "how to do it" question—but a form *of* experiential action that is increasingly responsive with the world (Kabat-Zinn, 1994, 2005). It is an act of attention one simply does. Attentiveness thus becomes both a means for cultivating responsiveness and an end in itself as a form of responsive action (Nhat Hanh, 1976). Indeed, this embodied understanding of attentiveness is central to distinguishing this form of awareness from other practices of introspective meditation or therapy, which typically require concentrated attention on a singular object of focused study in a controlled meditative environment (Depraz, Varela, and Vermesch, 2003).

Cultivating attentiveness is neither easy nor instantaneous, and requires on-going disciplined practice—over, and over, and over again. It requires that the process of attentiveness itself becomes simultaneously alert *and* habituated. It demands an intentional act, an intentional focus that cannot become habitual, yet which needs to be routinely practiced as a habituated act. That is, for attentiveness to be fostered, the perceptual muscles and senses involved in sharpening responsiveness must be trained to become habituated and the neuronal connections that support these sensory-motor couplings must be strengthened.

Promoting a culture of attentiveness in schools requires that teachers themselves exercise attentiveness in their teaching. Alert observation practiced attentively and routinely cultivates an embodied capacity for responsiveness that is required for the making of multiple—and potentially weak—metaphorical links among different conceptual domains. Teachers who offer provocations without dictating what will be learned, yet by simultaneously supporting and joining in multiple learning relations, provide the emotionally-embodied connections that extend learning. Such attentiveness opens up spaces for action that might contribute to promoting an ethic of multiplicity in the classroom, which might, as Mowat and Davis ask us to consider, strengthen neuronal networks by "ensuring concepts have metaphoric links to multiple source domains" (p. 28). This capacity to experience learning as multiple, dynamic constellations of relations offers a hope for an attentive quality in the mathematics classroom.

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