Canadian Studies in Population, Vol. 13 (1), 1986, pp. 83-102

RECTANGULARIZATION OF THE SURVIVAL CURVE AND ENTROPY: THE CANADIAN EXPERIENCE, 1921-1981

Dhruva Nagnur
Statistics Canada, Ottawa, Ontario, Canada

Résumé — Lorsqu'il est apparu dans les courbes de survie, le phénomène de la rectangularisation a été associé à l'accroissement de la probabilité de survie aux âges les plus avancés et à l'amélioration générale de l'espérance de vie. Ce qui s'est passé au Canada pendant les six dernières décennies offre une nouvelle preuve de l'existence de ce phénomène. Les valeurs de l'entropie (H) ainsi que la projection des espérances de vie totales à deux âges différents donnent la même 'durée de vie maximale moyenne', cette conclusion valent aussi bien pour les hommes que pour les femmes. Le présent document cherche également à exprimer H sous la forme d'un polynôme du deuxième degré dont l'inconnue est la réciproque de l'espérance de vie à différents âges successifs; la qualité de l'ajustement de la fonction est bonne. Enfin, l'importance de cet effet de rectangularisation pour quelques applications particulières liées aux soins de santé est rapidement analysée.

Abstract — The Emergence of the phenomenon of rectangularization of the survival curve is associated with the increased probability of survival to older ages and the general improvement of the expectation of life. The Canadian experience for the past six decades provides an added testimony to this phenomenon. The values of Entropy (H) as well as the projection of total life expectancies at two different age-points give the same 'average maximum life expectancy' for men and women. An effort to parameterize H with a second degree polynominal in the reciprocal of the expectation of life at successive ages is attempted. The function gives a good fit. The implication of the rectangularization with respect to specific health care areas is briefly examined.

Key Words — **rectangularization, survival curve, entropy, total life expectancy, average maximum life span**

Dhruva Nagnur

Introduction

In the recent demographic and epidemiologic literature, there has been an extensive discussion of the phenomenon of rectangularization of the life table survival curve (Fries, 1980, 1984; Keyfitz, 1977, 1978; Manton, 1982; and Myers and Manton, 1984). Rectangularization, or the flattening of the curve to form a more rectangular shape, has occurred over the past several decades in many developed countries of the world in which the demographic transition from high mortality to low mortality is nearly complete.

The emergence of the phenomemon of rectangularization of the survival curve has been the consequence of the increased probability of survival to older ages as a result of the improvement in mortality at all ages in general and at younger ages in particular. This experience has caused significant improvements in the expectation of life at birth and at successive ages, though the magnitude of such increase is found to be declining with age. Mention is also being made by some that the emergence of rectangularization of the survival curve may eventually be associated with the rectangularization of the morbidity curve, resulting in the compression of morbidity at older ages and thus contributing to an increase in the disease-free life at older ages. According to the evidence provided by hospital morbidity data in Canada, compression of morbidity does not seem to have taken place as yet. However, the Canadian mortality experience over the past six decades provides a significant testimony to the rectangularization phenomenon with its attendant increase in life expectancy and survivors to older ages.

Increasing Survivors to Older Ages

From 1921 to 1981, there have been significant increases in the probability of survival and the number of survivors at successively older ages (Table 1). These values are reproduced from a set of abridged life tables (Nagnur, 1986) constructed for five-year intervals from 1921 to 1981 for Canada and the provinces.

Generally, the improvements in the number of survivors were greater during the 30 years following 1951, compared to the similar period before. Women made greater gains than men and remarkable improvements have been registered in the probabilities of survival to older ages – 65, 75 and 85.

TABLE 1. SURVIVORS OF THE BIRTH COHORT OF 100,000 TO AGES 65, 75 AND 85, CANADA, 1921, 1951 AND 1981

Years \ Ages	Males			Females		
	65	75	85	65	75	85
1921	57,603	35,883	11,123	59,640	38,418	12,994
1951	65,877	41,931	13,490	75,585	54,046	20,760
1981	74,681	50,367	19,966	86,088	70,503	40,648
	Increase in the Number of Survivors					
1951-21	8,274	6,048	2,367	15,945	15,628	7,766
1981-51	8,804	8,436	6,476	10,503	16,457	19,888
Total	17,078	14,484	8,843	26,448	32,085	27,654
	Percent Increase in Survivors					
1951-1921	14.4	16.9	21.3	26.7	40.7	59.8
1981-1951	13.4	20.1	48.0	13.9	30.4	95.8

Source: Nagnur, Dhruva, 1986. Longevity and Historical Life Tables

Among the male birth-cohorts subject to the 1921 life table, three out of five were expected to survive to the age of 65; the proportion according to the 1981 life table had risen to three out of four. Slightly more than one out of three survived to age 75 according to the 1921 life table; while in the 1981 life table, more than one half were expected to make it to that age. A remarkable progress has occurred with respect to the improvements in the chance of survival to age 85. Between 1951 and 1981, the increse in the numbers expected to reach age 85 was almost three times as many as those between 1921 and 1951 — 6,500 as against 2,400 from an initial birth-cohort of 100 thousand.

With respect to women, the improvements in the corresponding survival probabilities have been still more spectacular than those for men. According to the 1921 life table, women who would survive to 65 years of age accounted for three out of five; the proportion had risen to three out

of four and was almost seven out of eight according to the 1951 and 1981 life tables. A similar trend was observed among women estimated to survive to age 75; according to the 1921 life table, two out of five were expected to survive to that age; the proportion had increased to more than one in two and it was in excess of seven out of 10 according to the 1951 and 1981 life tables.

Survival Curves

The survival curves, the number surviving to each age (*lx* column of a life table) plotted against the age, illustrate the phenomenon of rectangularization rather dramatically. The flattening of the curve is gradually evolving over the last six decades. Some significant developments in the Canadian Experience can be observed from the set of graphs in Charts 1 to 4. These charts illustrate the gradual evolution of the increasing number of survivors to older ages for both men and women from 1921 to 1981. The increase in the rectangularization is relatively greater during the period 1951 to 1981 than in the earlier three decades. The differential between men and women was more pronounced at the older ages in 1981 than in 1951 and was clearly very small in 1921. In any case, one can reasonably suggest that future movement towards rectangularization will be in much smaller increments than those for the past six decades. Fries states that, with respect to the U.S.A., 80 per cent of the difference between the historical curve (of the year 1900) and the ideal curve was eliminated by 1980; this may also be true for Canada. Of course, the ideal curve (terminating at about age 90) or complete rectangularization is a theoretical concept; it is unachievable in real life.

Trends in Life Expectancy

Life expectancy at birth has increased significantly during the past six decades — 13 years for men and 18 years for women. The primary factor contributing to this increase is the reduction of infant and childhood mortality. The infant mortality rate, which was in excess of one in 10 live births in 1921, has declined to less than one in 100 live births in 1981; the neonatal death rate, which was 43 per 1,000 live births in 1921, decreased to six per 1,000 live births in 1981; similarly, the post-neonatal death

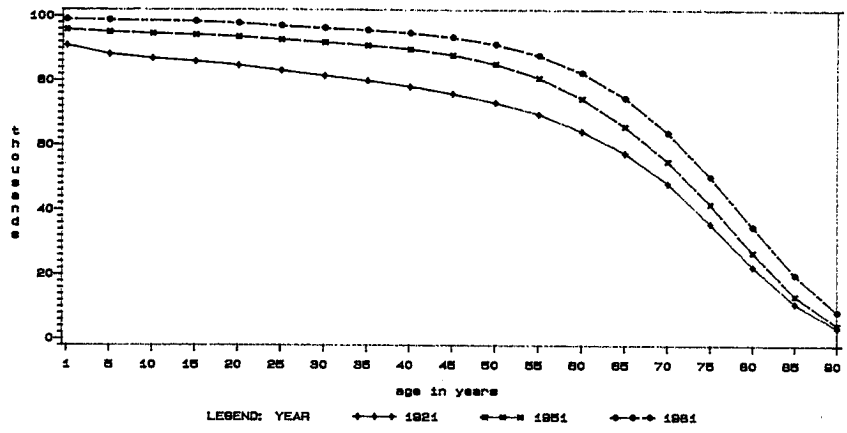

CHART 1. SURVIVORS OUT OF 100,000 BORN ALIVE BY AGE, CANADA: MALES, 1921, 1951 and 1981

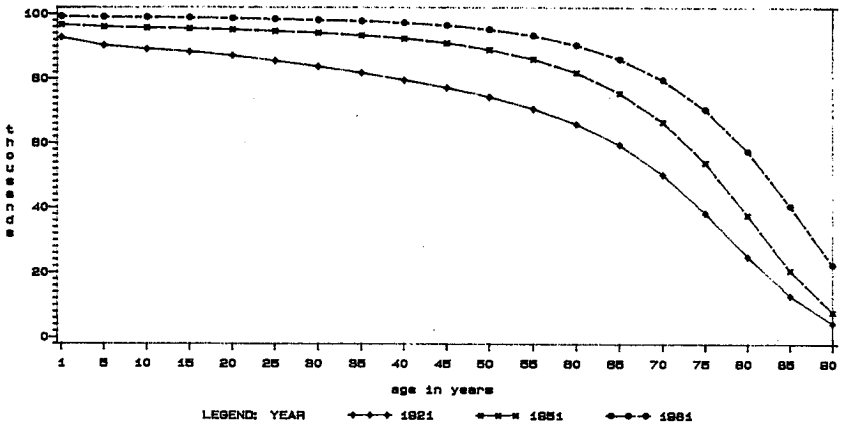

CHART 2. SURVIVORS OUT OF 100,000 BORN ALIVE BY AGE, CANADA: FEMALES, 1921, 1951 and 1981

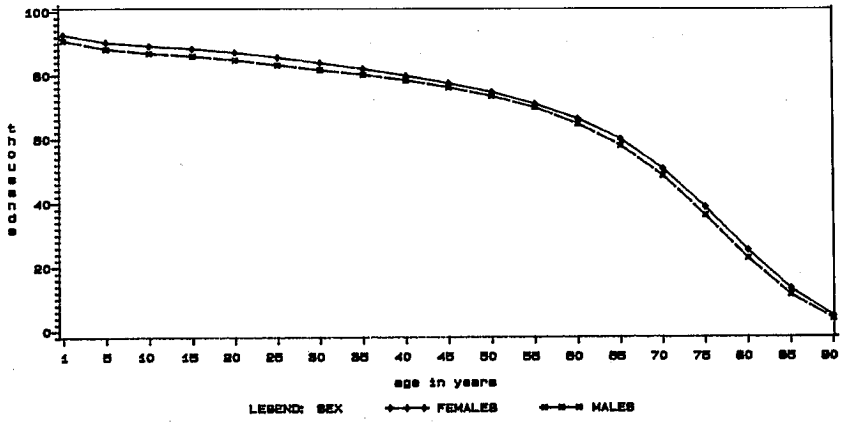

CHART 3. SURVIVORS OUT OF 100,000 BORN ALIVE BY AGE AND SEX, CANADA: 1921

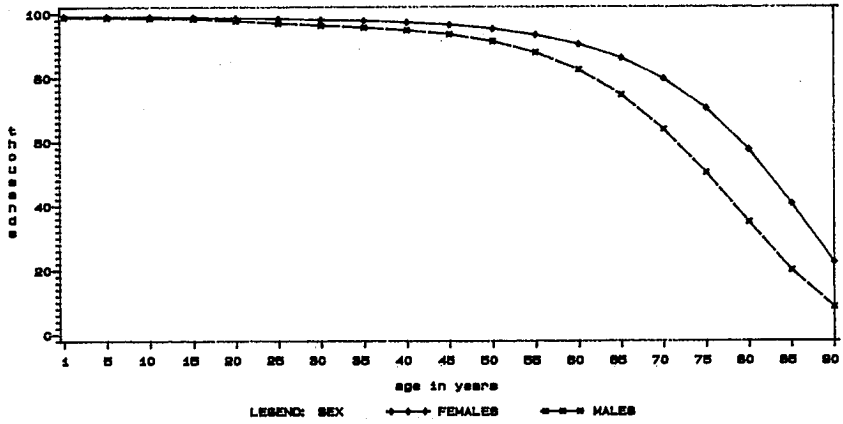

CHART 4. SURVIVORS OUT OF 100,000 BORN ALIVE BY AGE AND SEX, CANADA: 1981

rate, which was around 45 per 1,000 live births in 1921, fell to slightly more than three per 1,000 live births in 1981. Such dramatic declines have contributed proportionately more than those of any other single age to the increase in the life expectancy at birth.

It can be seen from Table 2 that there have been more improvements in the first three decades following 1921 than in the latter three decades.

TABLE 2. LIFE EXPECTANCY AT AGE 0, 40, 65 AND 75, CANADA, 1921, 1951 AND 1981

Age	Males			Females		
	1921	1951	1981	1921	1951	1981
0	58.8	66.4	71.9	60.6	70.9	79.0
40	32.1	32.5	34.7	32.7	35.7	40.8
65	13.0	13.3	14.6	13.6	15.0	18.9
75	7.6	7.9	9.0	8.0	8.8	11.9

Age	Increase in Life Expectancy					
	Males			Females		
	1921-51	1951-81	Total	1921-51	1951-81	Total
0	7.6 (12.9)	5.5 (8.3)	13.1 (22.3)	10.3 (17.0)	8.1 (11.4)	18.4 (30.4)
40	0.4 (1.2)	2.2 (6.8)	2.6 (8.1)	3.0 (9.2)	5.1 (14.3)	8.1 (24.8)
65	0.3 (2.3)	1.3 (9.8)	1.6 (12.3)	1.4 (10.3)	3.9 (26.0)	5.3 (39.0)
75	0.3 (3.9)	1.1 (13.9)	1.4 (18.4)	0.8 (10.0)	3.1 (35.2)	3.9 (48.7)

Age	Difference Between Female-Male Life Expectancy		
	1921	1951	1981
0	1.8	4.5	7.1
40	0.6	3.2	6.1
65	0.6	1.7	4.3
75	0.4	0.9	2.9

Figures in bracket show percentage increases to the beginning of the period.

Source: Nagnur, Dhruva. 1986. Longevity and Historical Life Tables

The sex difference in the expectation of life has increased steadily from about two years in 1921 to more than seven years in 1981.

Although the expectation of life at birth has increased substantially since 1921, the same is not true for life expectancy at the older ages — for instance, at 40, 65 and 75; this is relatively more so in the case of males than females. Charts 5 to 8 show this trend rather dramatically. Between 1921 and 1951, the life expectancy for men at ages 40, 65 and 75 increased by less than half a year. For women, the improvement, though progressively smaller with increasing age, was relatively more significant — three years at age 40, 1.8 years at age 65 and 0.8 year at age 75.

There were, however, comparatively greater improvements in expectation of life at older ages between 1951 and 1981. Men improved their life expectancies by 2.2, 1.3 and 1.1 years at ages 40, 65 and 75; women, by 5.1, 3.9 and 3.1 years.

The difference in life expectancy between men and women has steadily increased over the past six decades — from about two years more at birth for women in 1921 to more than seven years in 1981; the same difference at age 75 in 1981 has been less than three years. During the past 60 years, men have improved their life expectancy at birth by an average of 2.6 months per year while women have increased theirs by 3.7 months. At age 75, the average improvement for men has been 0.28 months per year; while for women it has been three times as much, 0.78 months.

Entropy

Another interesting and informative way of looking at the phenomenon of rectangularization is to observe the temporal variations in the values of H, the entropy, or information (Keyfitz, 1977). H_a is defined by Keyfitz as:

$$H_a = -\frac{\int_a^w l(x) \log l(x)\, dx}{\int_a^w l(x)\, dx}$$

where a is the initial age, w is the terminal age, and $l(x)$ is the life table survivor-column. As stated by Keyfitz, what the index H_a measures,

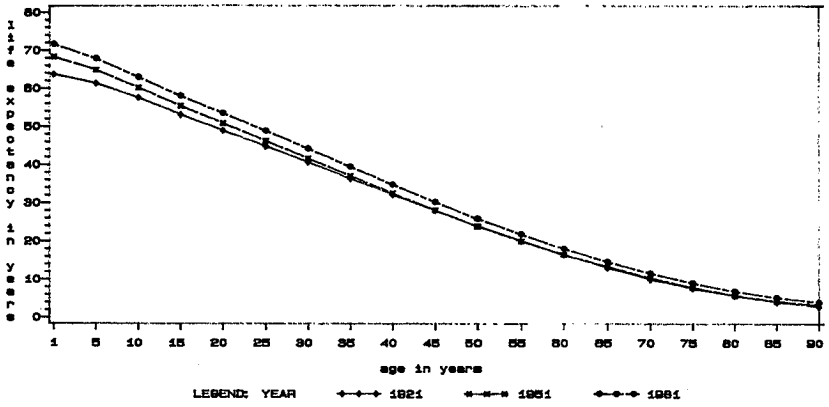

CHART 5. LIFE EXPECTANCY BY AGE, CANADA: MALES
1921, 1951 AND 1981

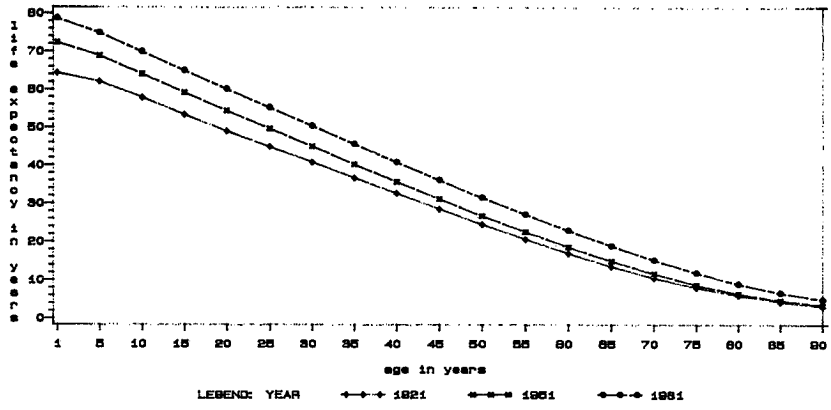

CHART 6. LIFE EXPECTANCY BY AGE, CANADA: FEMALES
1921, 1951 AND 1981

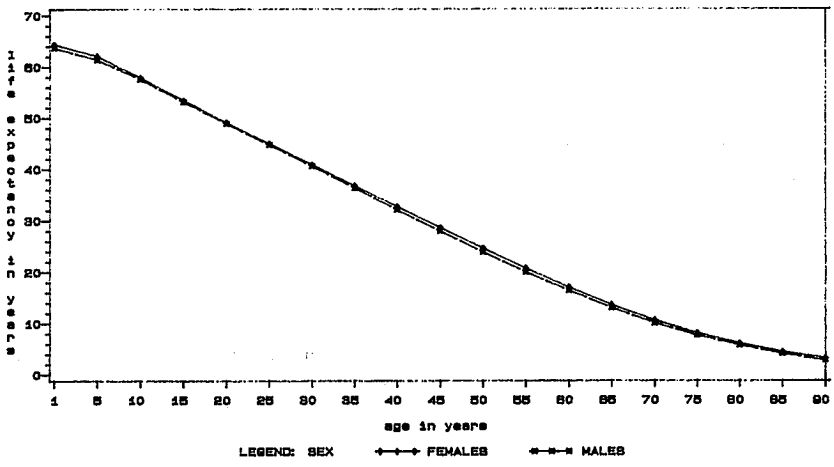

CHART 7. LIFE EXPECTANCY BY AGE AND SEX, CANADA: 1921

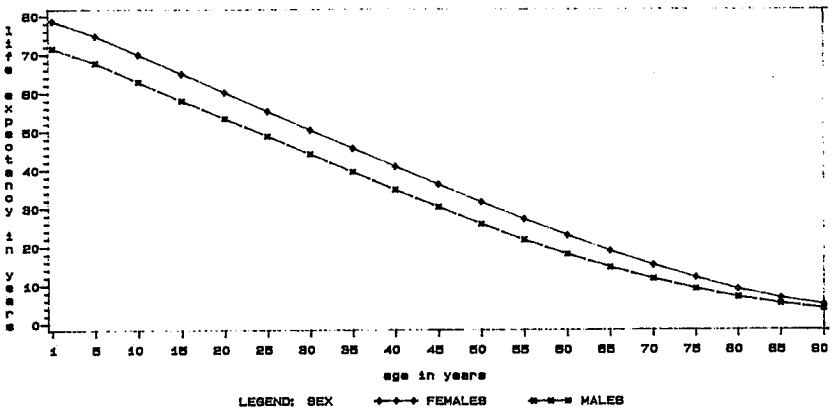

CHART 8. LIFE EXPECTANCY BY AGE AND SEX, CANADA: 1981

quantitatively, is the percentage improvement in the expectation of life at age a, if one per cent improvement in mortality, or age-specific death rates, takes place at all ages a and above.

As the number of survivors to older ages has increased and life expectancy at birth has improved over the years and decades in Canada, the value of H_0 (H at age 0) has decreased. Consequently, the contribution of the proportional improvement in the age-specific mortality rates on the expectation of life has diminished correspondingly.

Table 3 gives the values of H with respect to Canadian life tables for men and women for the years 1921, 1951 and 1981. In Charts 9 to 12, the values of H are plotted against age for the years 1921, 1951 and 1981 for both men and women. The curves in these charts present a sort of mirror image of the survival curves shown in Charts 1 to 4. The process of rectangularization may be observed clearly: the curve is being pulled towards the lower values of H at older ages as the proportion of survivors to older ages increases.

The curve of H against age, plotted for both men and women, obviously highlights the differences in the number of survivors at older ages. The H values are consistently smaller for women than for men — reflecting the relative difference, in favour of women, in the survival probabilities and life expectations at various ages.

TABLE 3. ENTROPY (H) VALUES, CANADIAN LIFE TABLES, 1921, 1951 AND 1981

	Males	e_0^0	Females	e_0^0
1921	0.302826	58.84	0.282605	60.60
1951	0.193278	66.41	0.161395	70.90
1981	0.144227	71.85	0.112732	79.04

Source: Nagnur, Dhruva. 1986. Longevity and Historical Life Tables

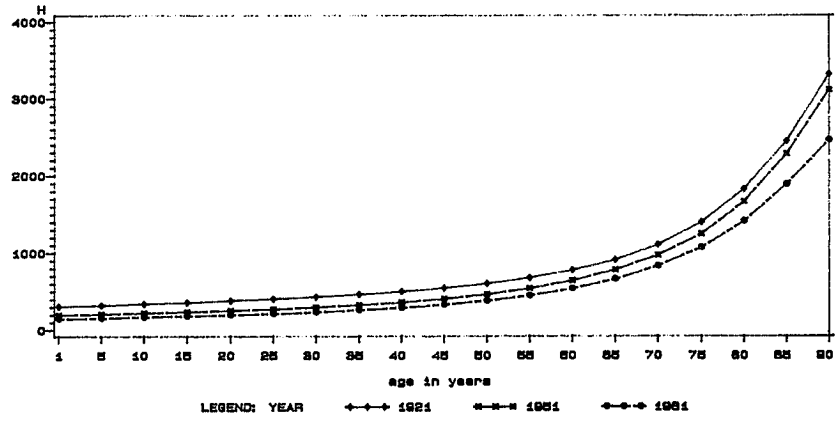

CHART 9. ENTROPY (*H*) BY AGE, CANADA: MALES
1921, 1951 and 1981

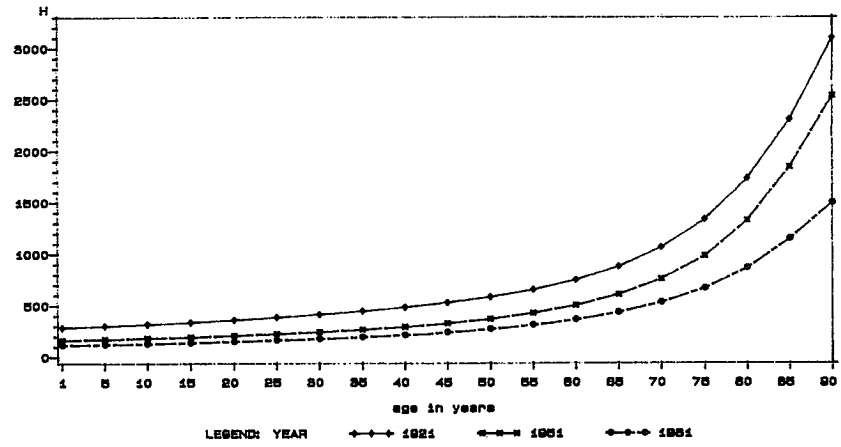

CHART 10. ENTROPY (*H*) BY AGE, CANADA: FEMALES
1921, 1951 and 1981

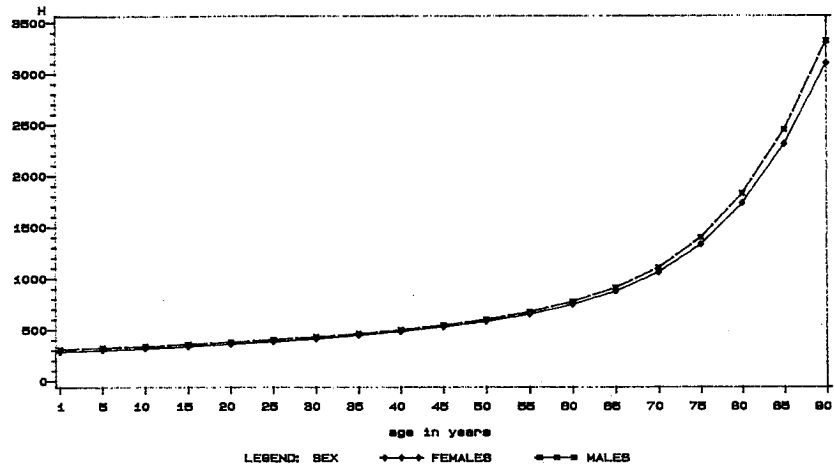

CHART 11. ENTROPY (*H*) BY AGE AND SEX, CANADA: 1921

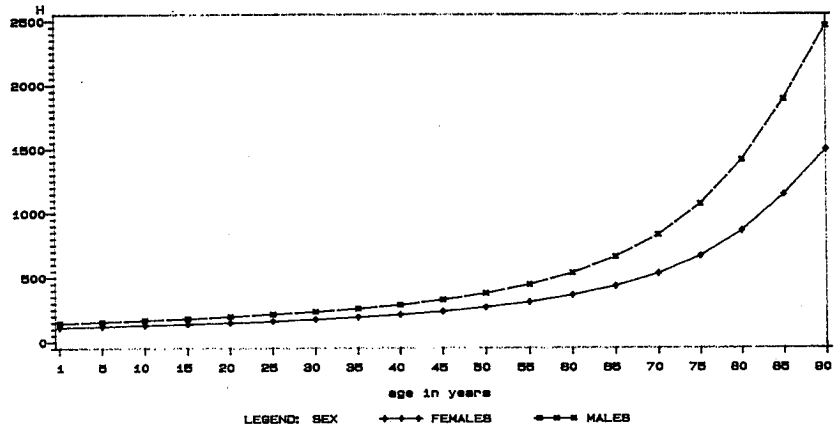

CHART 12. ENTROPY (*H*) BY AGE AND SEX, CANADA: 1981

Average Maximum Life Span

The concept and definition of H points towards a way to derive an approximate measure of an average maximum life span. If a one per cent decline in the age-specific mortality rates across the board increases the life expectancy by H per cent, one could argue that, at least theoretically, a 100 per cent elimination of the age-specific death-rates would improve life expectancy by 100 H per cent. This would approximately give the estimate of the average maximum life-span. Though it is tempting to put forth, it may not be correct to assume a straightforward relationship between the improvement of mortality and the increase in the expectation of life, since at a higher percentage of improvement in age-specific death rates, the approximations employed for H may not hold true. More than one method, however, could be employed to derive the estimation of average maximum life span. The following represents an illustration of a couple of possible approaches towards an attempt to derive estimates of average maximum life span. The results are no more than the consequence of the application of these approaches to Canadian data. Any interpretation will have to take this aspect into consideration.

For example, the H values at age zero for Canadian men and women for 1981 were 0.1442 and 0.1127 respectively. 100 H per cent would give the maximum improvement of 14.42 per cent for men and 11.27 per cent for women. Applying these percentages to the expectations of life at birth of 71.9 years for men and 79.0 years for women gives an estimate of the average maximum life span of 82.3 years for men and 87.9 years for women. A simple average for both sexes would give an estimate of 85.1 for the average maximum life span.

Another method to estimate the average maximum life span is to project the total life expectancies at two points and find out the value at the intersection (Fries, 1980).

If the total expectancies at ages 20 and 65 are projected on the assumption that the average of the increases for these ages for the past 60 years will continue in the future, the estimates of the average maximum life span for men and women turn out to be very close to that estimated earlier: 82.7 for men and 87.5 for women. The simple average for both sexes would be 85.1 years.

Following are the equations for deriving these values:

for men

$$\begin{cases} E_m = 73.40 + 0.075\ t \\ \\ E_m = 79.57 + 0.025\ t \end{cases}$$

and

for women

$$\begin{cases} E_m = 80.16 + 0.185\ t \\ \\ E_m = 83.93 + 0.090\ t \end{cases}$$

where t is the year and E_m the maximum life span.

Charts 13 and 14 show that the curves at age 20 and 65 intersect at about the year 2021 for women and 2105 for men.

Almost identical results are obtained for the average maximum life span for Canadian men and women using H or the method of projected total life expectancies (at ages 20 and 65). Other assumptions about future rates of increase in total expectancies with different pairs of age groups will yield different estimates of the average maximum life span; however, these estimates seem to vary within a narrow range. This may be important and worth taking into consideration in projecting life expectancies or mortality in the future.

Fitting of Curve to H

Since H is extremely important as an index for measuring the rectangularization, an attempt was made to fit a curve to the values of H against the expectation of life at different ages. Several functions were tried, and the following offered the best fit:.

$$H_x = A + B\left(\frac{1}{E_x}\right) + C\left(\frac{1}{E_x}\right)^2$$

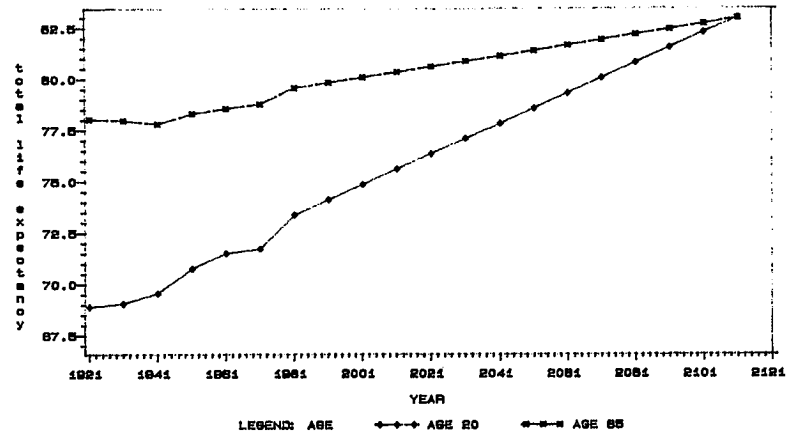

CHART 13. TRENDS AND CONVERGENCES IN LIFE EXPECTANCY, CANADA: MALES

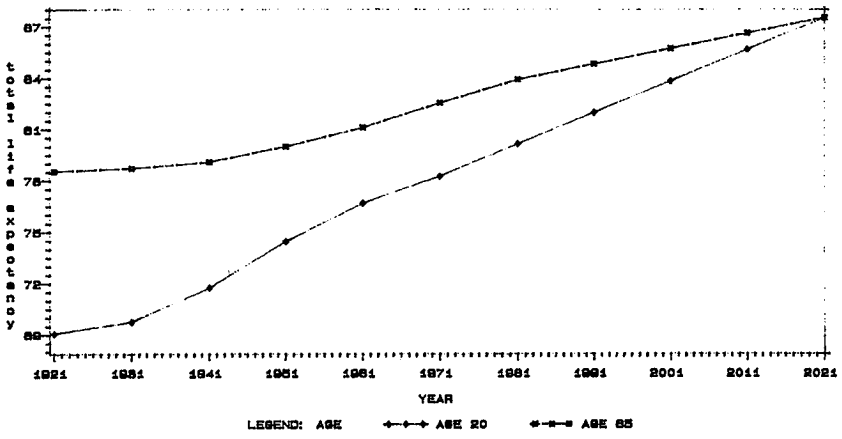

CHART 14. TRENDS AND CONVERGENCES IN LIFE EXPECTANCY, CANADA: FEMALES

Table 4 shows the fitted and the observed values of H for Canada for men and women, 1980-82. The fitting of the curve was performed by the Method of Least Squares. The implication of the function and the importance of the parameters are currently being examined for further analysis and in the context of life table modelling.

TABLE 4. FITTED AND OBSERVED H VALUES FOR MEN AND WOMEN, CANADA, 1980-1982

Canada, Males, 1980-1982

A = 0.011700 B = 9.510665 C = 0.395607

Fitted equation is H_x = A + B(1/EX)+C(1/EX)**2

Canada, Females, 1980-1982

A = 0.013842 B = 8.084172 C = -4.153461

Fitted Equation is H_x = A + B(1/EX)+C(1/EX)**2

Observed H	Estimated H	EX	Age	Observed H	Estimated H	EX
0.144227	0.144145	71.85	0	0.112732	0.115456	79.04
0.146247	0.144552	71.63	1	0.114168	0.115992	78.70
0.154238	0.152062	67.80	5	0.119823	0.121119	74.84
0.165450	0.162979	62.91	10	0.127663	0.128596	69.93
0.178431	0.175710	58.03	15	0.136609	0.137212	65.01
0.193609	0.189975	53.39	20	0.146932	0.147094	60.15
0.211123	0.206797	48.79	25	0.158895	0.158723	55.28
0.231576	0.227516	44.11	30	0.172968	0.172544	50.42
0.256204	0.253404	39.39	35	0.189793	0.189205	45.58
0.286765	0.285953	34.72	40	0.210286	0.209535	40.79
0.325647	0.327371	30.17	45	0.235624	0.234593	36.10
0.376252	0.380782	25.81	50	0.267460	0.266060	31.53
0.443304	0.450213	21.73	55	0.308165	0.306178	27.13
0.533618	0.542178	17.97	60	0.361130	0.358648	22.92
0.657131	0.666321	14.57	65	0.431613	0.429307	18.93
0.828696	0.835952	11.58	70	0.528079	0.526741	15.23
1.069610	1.073325	9.00	75	0.663794	0.665422	11.87
1.411273	1.404459	6.87	80	0.860072	0.865249	8.95
1.885025	1.862480	5.18	85	1.142817	1.141801	6.61
2.457720	2.469972	3.91	90	1.494102	1.493349	4.89

Implications and Conclusions

The continuous and differential rectangularization of the survival curve has significant implications for Canadian society in terms of health care utilization, provisions for the old and the very old, homes for the aged, recreation facilities for the elderly, shifts of population to warmer climates and so on. For example, with respect to one aspect of health care services — the hospital in-patient service — the hospital morbidity data for Canada for the past two decades throw important light on the directions and trends in utilization (Table 5).

The population for both males and females increased, during the period 1961-81, by 70 and 76 per cent for the age groups 65+ and 75+. While the hospital in-patient days increased by 132 and 162 per cent respectively for the age groups 65+ and 75+, with respect to all age groups, the increase in the patient days was 44 per cent. The average number of hospital days per person increased by 36.5 per cent for those 65+ and 48 per cent for those 75+. The number of cases of separations from the hospital increased by 124 and 131 per cent for the age groups 65+ and 75+ respectively. The average number of hospital days per separation registered in 1981 was 26 days for those 65+ and 33.5 days for those 75+.

The increasing number of elderly as a consequence of the increased rectangularization is likely to be associated with greater utilization of health care facilities and services. This would have important societal consequences in the long run, especially if it is not accompanied by a corresponding compression of morbidity — the compression of morbidity is equivalent to the prolongation of the disease-free state at the older ages so that the period of ill health prior to death is significantly minimized.

Moreover, as degenerative diseases like cardiovascular and cancer are treated successfully and effective preventive measures to eliminate or postpone the onset of these diseases are established, it is likely that more and more people will tend to live longer and perhaps healthier lives. This affects a whole set of social concerns in addition to the health care areas, such as retirement age, second careers or career after retirement. The phenomenon of increasing rectangularization along with other emerging demographic and epidemiologic developments provides an important prospective look at the future population profile and emerging societal concerns.

TABLE 5. POPULATION AND HOSPITAL MORBIDITY STATISTICS FOR THE AGE GROUPS 65+ AND 75+, CANADA, 1961 AND 1981

		Age Groups		Total All Age Groups
		65+	75+	
Population (in thousands)	1961	1,390.2 (7.6)	501.6 (2.8)	18,200.7 (100.0)
	1981	2,359.2 (9.7)	882.8 (3.6)	24,274.3 (100.0)
Percent Increase 1961-1981		69.7	76.0	33.4
Hospital Patient Days (millions)	1961	8.8 (29.6)	4.9 (16.5)	29.7 (100.0)
	1981	20.4 (47.8)	12.8 (30.0)	42.7 (100.0)
Percentage Increase 1961-1981		131.8	161.2	43.8
Average No. of Hospital Days (per person)	1961	6.3	9.8	1.6
	1981	8.6	14.5	1.8
Percentage Increase 1961-1981		36.5	48.0	12.5
Hospital Separations (in thousands)	1961	353.2 (13.0)	165.7 (6.1)	2,726.5 (100.0)
	1981	790.8 (22.2)	382.4 (10.7)	3,570.0 (100.0)
Percentage Increase 1961-1981		123.9	130.8	30.9
Average Hospital Days per Separations	1961	24.9	29.6	10.9
	1981	25.8	33.5	12.0
Percentage Increase 1961-1981		3.6	13.1	10.1
Average Number of Separations per 100 Persons	1961	25	33	15
	1981	34	43	15
Percentage Increase 1961-1981		36.0	30.3	0.0

Source: C. Nair, personal communication and information from Hospital Morbidity, Cat. 82-206, Statistics Canada.

Acknowledgments

The author wishes to express his deep appreciation to Dr. Paul Reed for support and encouragement. Thanks are also extended to Marie-Claire Couture, Ted Wannell and Audrey Sirois for their considerable assistance in the preparation of the manuscript.

Disclaimer

The analysis is the responsibility of the author and does not necessarily represent the views of Statistics Canada.

References

Fries, J.F. 1980. Aging, natural death, and compression of morbidity. The New England Journal of Medicine 303:130-135.

_____. 1984. The Compression of morbidity, miscellaneous comments about a theme. The Gerontologist 24(4):354-3359.

Keyfitz, N. 1977. Applied Mathematical Demography. New York:John Wiley and Sons.

_____. 1978. Improving life expectancy, and uphill road. American Journal of Public Health 68:954-956.

Nagnur, D. 1986. Longevity and Historical Life Tables, Canada and Provinces, 1921-1981. Catalogue 89-506. Ottawa, Ontario: Statistics Canada.

Manton, K.G. 1982. Changing concepts of mortality and morbidity in the elderly population. Milbank Memorial Fund Quarterly 60(2):183-244.

Myers, G. and K. Manton. 1984. Compression of mortality – myth or reality. The Gerontologist 24(4):346-353.

Statistics Canada. 1984. Hospital Morbidity. Catalogue 82-206. Ottawa, Ontario: Supply and Services Canada.

Received May, 1985; January, 1986.