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CANADIAN KINSHIP PATTERNS BASED ON 1971 AND 1981 DATA*

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Résumé —Liens de parenté selon les donnés de 1971 et 1981. La réduction de la fécondité produit non seulement moins d'enfents dans la famille, mais aussi moins de parenté. Les taux de naissances au Canada ont aussi changé la structure de parenté: moins de soeurs, beaucoup moins de neveux et de nièces. Une réduction considérable dans le nombre de parents est implicite dans les taux de naissance et de décès de 1981, en comparaison avec 1971. Nous traçons, en graphiques et tableaux, les changements dans le nombre moyen de parents à travers le cycle de la vie. A cause du nombre réduit de parents, la famille ne peut presque plus remplir ses fonctions traditionnelles.

Abstract — Modern low fertility results not only in fewer children in the nuclear family, but fewer kin of all degrees. The lower births now prevailing in Canada also imply changes in the kin structure: fewer sisters of Ego, and more than proportionately fewer nephews and cousins. A major shrinkage of kin is implied by 1981 births and deaths in contrast to those of 1971. Curves and tables trace the changes in Ego's average number of kin through life. Small numbers of kin make it difficult for the family to fulfil its traditional functions.

Key Words — **kinship, stable populations, extended family, fertility**

*Data provided by Dhruva Nagnur of Statistics Canada

When the birth rate falls, as it did in Canada from 1971 to 1981, not only is the number of children in the average family smaller, so that people have fewer brothers and sisters, but they also have fewer aunts, nephews, cousins and other kin. This change in the size of the extended family affects its ability to support its older members. Beyond material support, traditional social patterns change as birth rates fall; kin give way to other groupings, partly because fewer kin are present. On the one side social change influences the birth rate, and on the other side the birth rate, through diminishing kin numbers, causes social change.

Given the difficulties of social security in advanced societies, it would seem sound policy to turn to the family for at least moral, if not material, support of the old. Yet, this runs into the obstacle that the same slowdown of births that causes aging and hence the crisis of social security also reduces the kin group that could supplement social security. The present investigation attempts to see how the numbers of kin are likely to change as a result of the continuing low number of births; it does so by comparing kin on the 1981 Canadian demographic regime with those on the regime of 1971.

This paper attempts to measure the size of kin group implied by the Canadian birth rates of 1971 and 1981. Of the various ways of measuring the numbers of kin, the most straightforward is that of comparative statics: the comparison of two stable conditions. The only data required are age-specific rates of birth and death; a series of formulas involving integration of familiar demographic variables permits calculation of the number of children, aunts, cousins, etc. implied by the regimes of birth and death. Formulas have been worked out for some 26 different kin relations (Goodman, Keyfitz and Pullum, 1974); in the present paper these are applied to Canada in 1971 and 1981. Noreen Goldman (1984) used the method to compare kinship in the Republic of Korea between the late 1950s and the late 1970s, where also there was a sharp downturn in birth rates. Useful theory on this matter is due to the work of Le Bras (1973).

The right way to interpret such results is in an "If...then..." mode. Thus, *if* the birth and death rates for Canada over a period of time continue as they were in 1981, *then* a woman aged 40 would have 0.585 living nieces and 0.630 female cousins in the female line (Tables 6 and 10). This is in contrast to 1971, whose rates correspond to 0.899 nieces and 0.983 female cousins. The difference of 20 per cent in births between the two dates is amplified in the different kin numbers implied.

Thus, in general, the recent fall of fertility means not only fewer children but just as surely fewer uncles, aunts, nieces, cousins and other kin. As fertility goes below the level of bare replacement, the kin networks shrink, most sharply for kin of a younger generation. The small size of the extended family would prevent its being a corporation for economic or defence purposes, even if there were no other reasons for its decline in importance.

The numbers in the tables and charts presented here derive from the fact that the regime of births and deaths directly determines average kin. If one thinks of a girl just born, and that at each year (or each moment) she has a certain probability of dying, and within certain ages (moment by moment) a certain probability of bearing a child, then a calculation which in effect follows her through life will give the expected number of her children. And if each of the children in turn is followed through life, similar calculations will give the expected number of grandchildren of the original girl. If the same process is carried out for each of the siblings of the girl in question, then the average number of nieces will be obtained, and similarly the process can be applied to other kin.

One could alternatively work out individual genealogies. Each woman will realize a certain age at death, and during her reproductive life a certain number of children will have been born. If these realizations are calculated by applying random numbers at each age according to the probabilities shown for the population of which the person is a member, and the process is carried out over several generations, and independently for each progenitor of an (imagined) population, then the total or average of these genealogies will be another way of getting at the average kinship. The simulation can be performed with weaker assumptions than are needed for the expected values obtained in this paper, and it is hoped that it will be used to see the impact of our (somewhat strong) assumptions. Simulation can provide variances as well as means.

A further step in the direction of realism is actual survey. In principle a census could be taken in which each person is asked how many siblings, cousins, and so on , he or she has. If the object is to describe the existing population, no theoretical calculation can substitute for an enumeration on the ground. Our numbers might conceivably be regarded as attempts to fill the lack of data, but they are actually something quite different: the implications for kinship over a long period in which the assumed birth and death rates are maintained.

Descendants Born and Living

Table 1 and Figure 1 show the curve of girl children born that, on this one-sex model, rises towards the usual gross reproduction rate and reaches it somewhat after age 40. Thereafter, there can be no change; the curves become horizontal, at the level of 0.813 for 1981 and 1.036 for 1971. There

TABLE 1. NUMBER OF LIVING DESCENDANTS

	DAUGHTERS BORN		DAUGHTERS ALIVE	
AGE	1981	1971	1981	1971
0	0.000	0.000	0.000	0.000
20	0.063	0.094	0.063	0.091
40	0.804	1.012	0.795	0.989
60	0.813	1.036	0.797	1.001
80	0.813	1.036	0.776	0.949
	GRANDDAUGHTERS BORN		GRANDDAUGHTERS ALIVE	
AGE	1981	1971	1981	1971
0	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.000
40	0.019	0.038	0.019	0.037
60	0.509	0.794	0.503	0.774
80	0.649	1.039	0.639	1.009

are no surprises here or in the next column showing children living as a descending curve after age 45. The rate of descent for 1971 corresponds to its longevity level, with an expectation of life at age zero of 80.8 years for 1981 and 76.4 for 1971. Even though the mortality improvement was less than five years, the difference between 1971 and 1981, especially for granddaughters, is striking.

This paper, like much other demographic work, confines its calculations to the female side of the population. The results for males would be similar, requiring only the entry of male mortality and fertility. The higher male mortality, and fertility spread over a wider range of ages, would have an appreciable though not large effect.

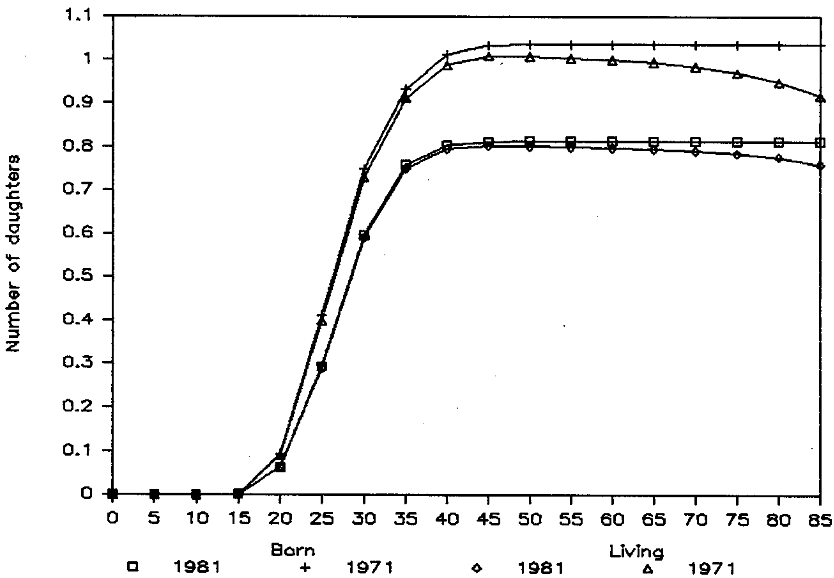

FIGURE 1. CHILDREN BORN AND LIVING
CANADA 1981 AND 1971

Ancestors Surviving

Table 2 and Figure 2 compare the survivorship of ancestors. The two top curves of the figure are for mothers, and both start at unity, since the mother must be alive at the birth of her child. (The father need only be alive nine months earlier, and this makes a certain difference in the calculations, not elaborated upon here.) The wide difference between the two top curves of Figure 2 again reflects mainly the 1981 expectation of 80.8 years against 76.4 for 1971.

Survivorship of ancestors might be expected to be affected solely by the death rate assumed, but this is not quite so. The birth rate is also involved. When the rate of increase of a population is rapid, mothers are younger on the average than when the rate of increase is slower, and hence, with a given

TABLE 2. PROBABILITY OF LIVING MOTHER, GRANDMOTHER AND GREAT-GRANDMOTHER

	MOTHER		GRANDMOTHER	
AGE	1981	1971	1981	1971
0	1.000	1.000	0.960	0.926
20	0.983	0.967	0.756	0.658
40	0.889	0.808	0.161	0.147
60	0.396	0.322	0.000	0.000
80	0.000	0.000	0.000	0.000
	GREAT GRANDMOTHER		GREAT–GREAT GRANDMOTHER	
AGE	1981	1971	1981	1971
0	0.554	0.481	0.024	0.029
20	0.063	0.065	0.000	0.000
40	0.000	0.000	0.000	0.000
60	0.000	0.000	0.000	0.000
80	0.000	0.000	0.000	0.000

death rate, the higher the number of births the higher the proportion of mothers surviving. This is at the population level, not at the individual level; a woman does not increase her longevity by having more children!

The bottom two curves of Figure 2 are for grandmothers, and of course they start out at different points – the chance that the grandmother is alive at the birth of the child (strictly when the child is under five years of age) is 0.960 for 1981, against 0.926 for 1971. On the average, grandmothers start to reach the last ages of life when the child is about 40, and the curves fall to zero just about there. Almost irrespective of the regime of mortality, few of us have living grandmothers once we pass age 40.

Table 2 also gives results for great-grandmothers and great-great-grandmothers. For the former, the chance of being present at the time the child is born is 0.554 for 1981. Great-great-grandmothers at 0.029 are slightly lower

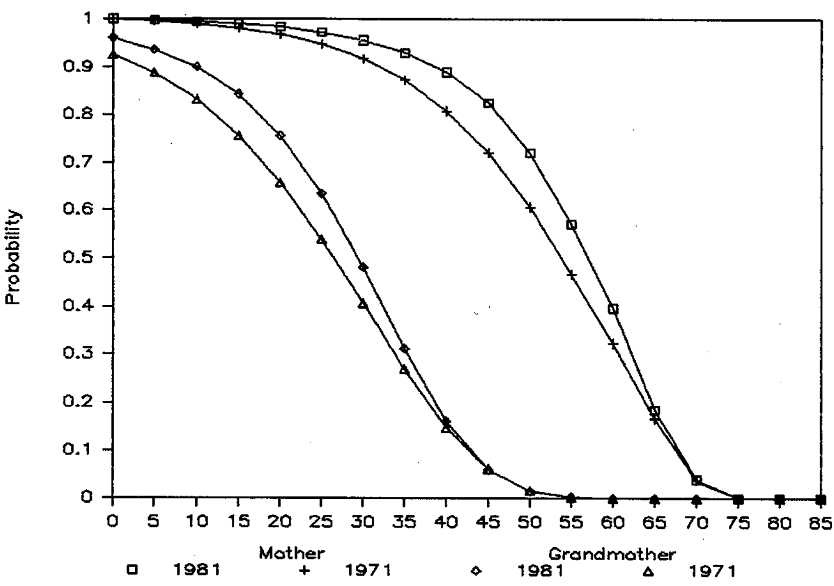

FIGURE 2. LIVING MOTHER, GRANDMOTHER
CANADA 1981 AND 1971

for 1981 than for 1971, despite improving mortality; this must be due to 1971's faster rate of increase that makes parenthood average slightly earlier in each generation.

Siblings

With Table 3 and Figure 3 we move from direct descendants and ancestors to collateral kin. The average number of older sisters born for 1981 is 0.415 and for 1971 0.516, corresponding to the 1.036/0.813 ratio in the birth rates of Table 1. Since one can never acquire additional older sisters after one is born, the graphs are perfectly horizontal. One can, on the other hand, acquire younger sisters, of whom one has none at the start, so these latter curves

TABLE 3. NUMBER OF SISTERS EVER BORN

	OLDER SISTERS		YOUNGER SISTERS		TOTAL SISTERS	
AGE	1981	1971	1981	1971	1981	1971
0	0.415	0.516	0.000	0.000	0.415	0.516
20	0.415	0.516	0.393	0.507	0.808	1.023
40	0.415	0.516	0.396	0.517	0.811	1.033
60	0.415	0.516	0.396	0.517	0.811	1.033
80	0.415	0.516	0.396	0.517	0.811	1.033

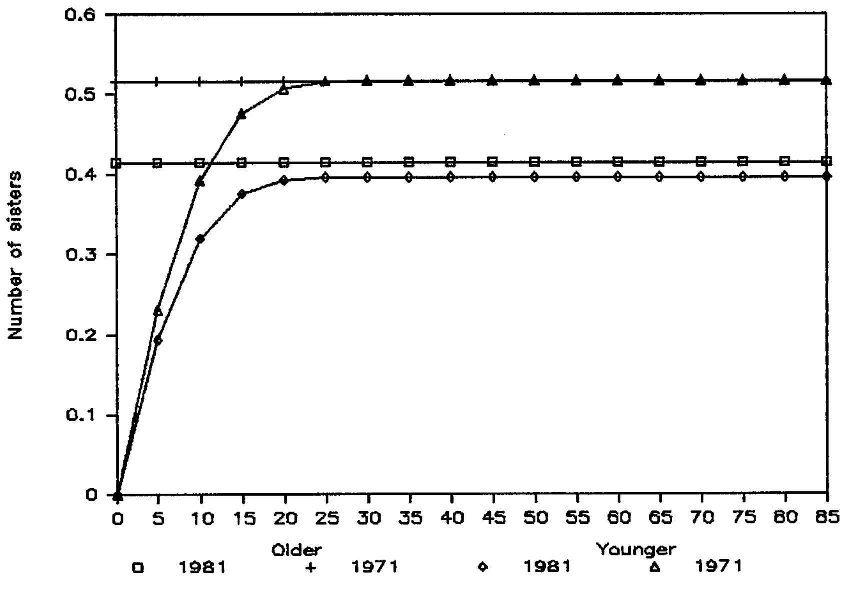

FIGURE 3. NUMBER OF SISTERS EVER BORN
CANADA 1981 AND 1971

rise steadily until they reach an asymptote at about age 25. Few persons acquire further siblings from the same mother after that age.

For 1971, the ultimate level for younger sisters is about the same as for older (0.517 against 0.516). For 1981, younger sisters are fewer than older. In a stationary population, on the average, people have the same number of younger and older siblings; in a decreasing population they have fewer younger than older. The difference was first discovered and applied to estimating fertility by Noreen Goldman (1978).

In a rapidly increasing population, there are more younger mothers than older mothers, all within the childbearing ages, as compared with a population growing more slowly. Goldman's way of using this to estimate the rate of increase of a population is based on the simplest of data: a survey in which women are asked only how many older sisters and younger sisters they have ever had (i.e., not taking account of deaths). McDaniel and Hammel (1981) have extended the method, using even less data: on their formula, the survey anthropologist asks each woman only if she is the oldest of her sorority, the youngest or in between, and surprisingly they obtain an estimate of the rate of increase the variance of which is not much greater than that for when all sisters are inquired upon.

For sisters surviving (Table 4 and Figure 4), we lost the horizontal asymptote, since beyond about age 25 one must expect a diminishing number of siblings, whatever the regime of fertility and mortality. The curves start at the same point as those of Table 3, but then decline, more rapidly for 1971 than for 1981.

Figure 4A shows the total of younger plus older sisters, and as one would anticipate, the curves for those born are horizontal after about age 20 or 25 – few siblings are separated in age by as much as 20 years – while the curves for those surviving gradually fall. Because of the difference in mortality, by the end of life the 1971 siblings have fallen nearly to the level of 1981.

TABLE 4. NUMBER OF SISTERS ALIVE

	OLDER SISTERS		YOUNGER SISTERS		TOTAL SISTERS	
AGE	1981	1971	1981	1971	1981	1971
0	0.411	0.502	0.000	0.000	0.411	0.502
20	0.409	0.502	0.388	0.496	0.797	0.997
40	0.402	0.486	0.389	0.500	0.791	0.985
60	0.367	0.408	0.378	0.474	0.745	0.883
80	0.169	0.162	0.319	0.353	0.489	0.515

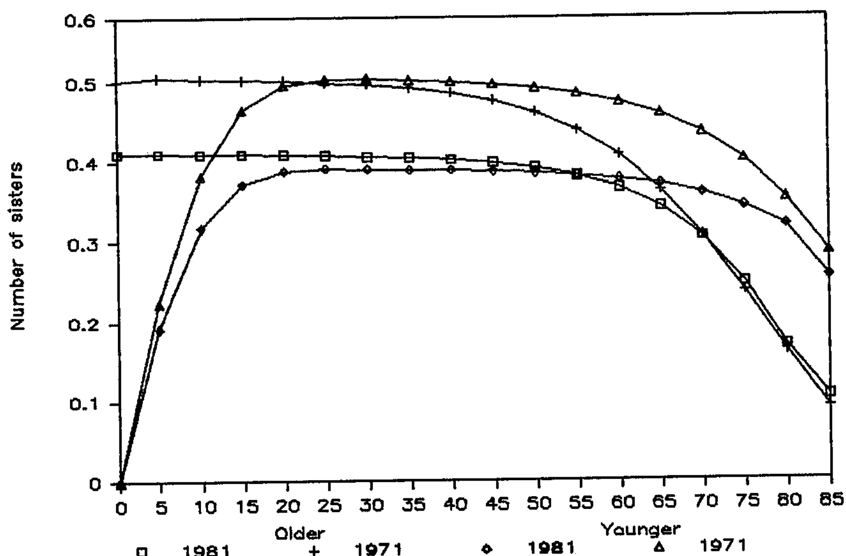

FIGURE 4. NUMBER OF SISTERS LIVING, CANADA 1981 AND 1971

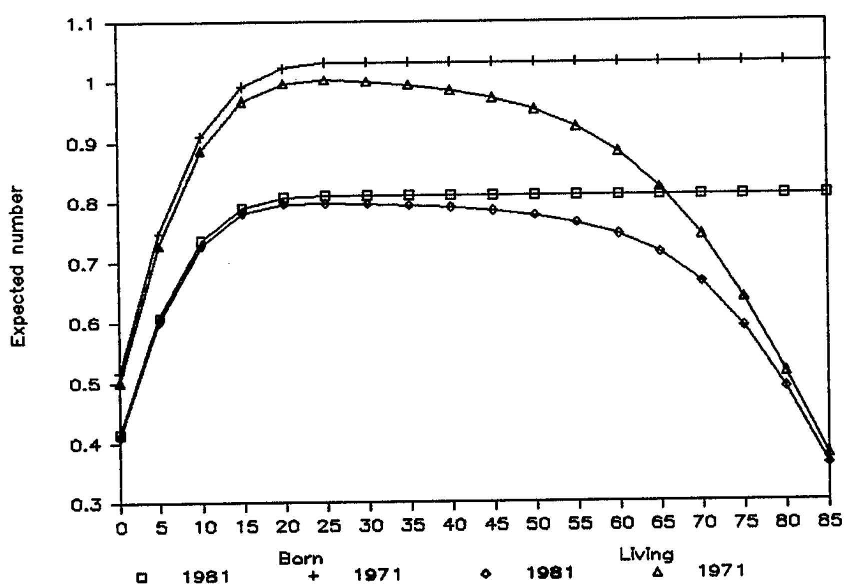

FIG. 4A. TOTAL SISTERS BORN AND LIVING, CANADA 1981 AND 1971

We seem to have a problem here. In Table 1 we saw that the ultimate number of daughters was 0.813 for 1981, and yet in Table 3 we find that the ultimate number of sisters ever born, adding younger and older, is 0.811. If we add to the 0.811 the quantity 1.0 for the girl in question, we find 1.811 for the number of girl children in the family, nearly one more than the 0.813 of Table 1. How can the number of daughters be 0.813 when we look at the matter from the viewpoint of the mother, and 1.811 when we look at it from the viewpoint of any one of the daughters?

The answer becomes clear, if we imagine gathering such data by a sample survey. A random sample of parents gives one result; a random sample of children is bound to give a higher result, since the representation of families would be directly proportional to the number of their children. Couples with no children would be included in the average when we ask parents; such couples would not appear at all when we ask children how many siblings they have. While our tables are not based on a survey, the computation gives the equivalent of a survey in this respect. It can be shown (Goodman, Keyfitz and Pullum, 1975) that under a Poisson distribution of children, with a fixed probability of having a child applied independently for all parents, and if every individual stands the same independent chance of being selected, the difference between sampling all families with equal probability and sampling all children with equal probability is exactly one person. The distribution implied in our work is not Poisson, but it is very nearly so.

Nieces

We are prepared to find that nieces born to older sisters in 1981 are more numerous than those born to younger sisters, since in part this reflects the larger number of older sisters in a population where deaths exceed births. In fact, for 1981 (Table 5) the nieces to older sisters are ultimately 0.332 as against 0.317 to younger sisteers, a ratio 1.048 of exactly the same amount as we found for sisters. The ratio for sisters comes to be reproduced for nieces. But that should not be surprising, since random variation is excluded from this work, which operates as though expected values always exactly materialize, and each mother has exactly the same number of children.

Notice that 1971 stood at 1.274 times 1981 on children born (Table 1), whereas it is 1.601 times 1981 in respect of nieces (Table 5). The ratio for nieces has to be higher because on the average the person has more siblings – and each sibling has more children – on the 1971 rates than on the rates for 1981. If in 1971 each mother averages 1.274 times 1981, multiplying this

TABLE 5. NUMBER OF NIECES EVER BORN

AGE	NIECES THROUGH OLDER SISTERS 1981	1971	NIECES THROUGH YOUNGER SISTERS 1981	1971	TOTAL NIECES 1981	1971
0	0.002	0.004	0.000	0.000	0.002	0.004
20	0.152	0.255	0.006	0.011	0.158	0.266
40	0.331	0.516	0.261	0.406	0.592	0.922
60	0.332	0.519	0.317	0.519	0.649	1.038
80	0.332	0.519	0.317	0.520	0.649	1.039

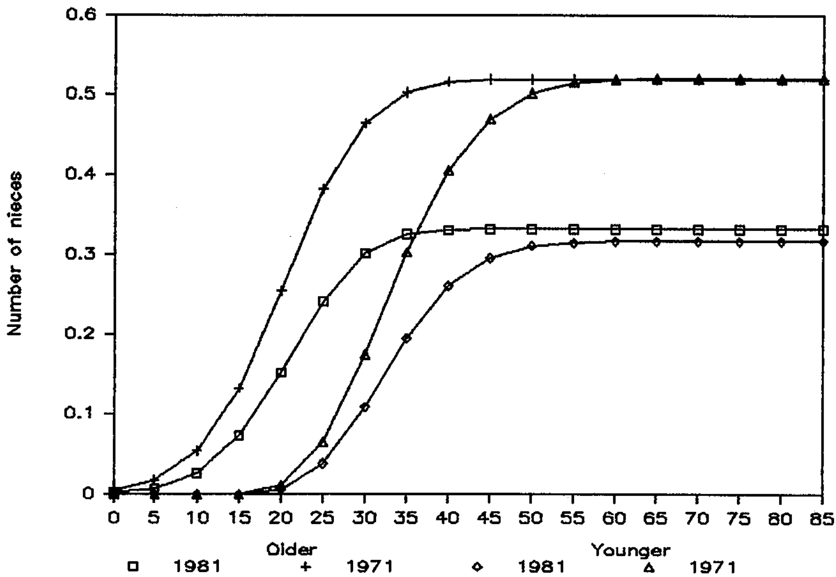

FIGURE 5. NUMBER OF NIECES EVER BORN
CANADA 1981 AND 1971

by the ratio for sisters, which is 1.033/0.811 = 1.274 again, we get 1.623, or only slightly more than the 1.601 above.

Table 6 shows that a person reaches the ultimate number of his or her nieces through a younger sister a little after age 40, and through an older sister be-

TABLE 6. NUMBER OF NIECES ALIVE

	NIECES THROUGH OLDER SISTERS		NIECES THROUGH YOUNGER SISTERS		TOTAL NIECES	
AGE	1981	1971	1981	1971	1981	1971
0	0.002	0.004	0.000	0.000	0.002	0.004
20	0.151	0.248	0.006	0.010	0.157	0.259
40	0.327	0.503	0.258	0.396	0.585	0.899
60	0.324	0.496	0.311	0.504	0.635	1.000
80	0.307	0.445	0.306	0.489	0.613	0.934

fore age 40. One's older sisters on the average end their childbearing sooner than one's younger sisters. Nieces through older sisters show the same curve in relation to nieces through younger sisters as that shown in Figure 3 for the sisters themselves.

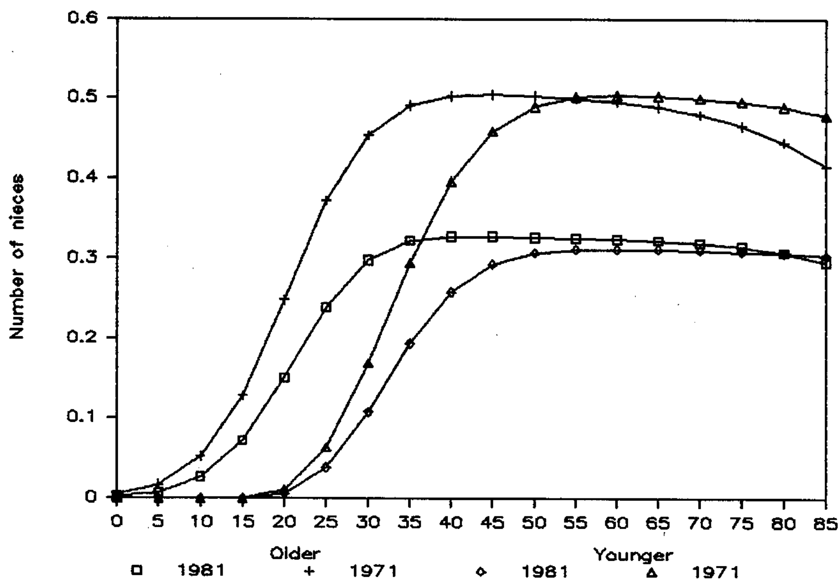

FIGURE 6. NUMBER OF NIECES LIVING
CANADA 1981 AND 1971

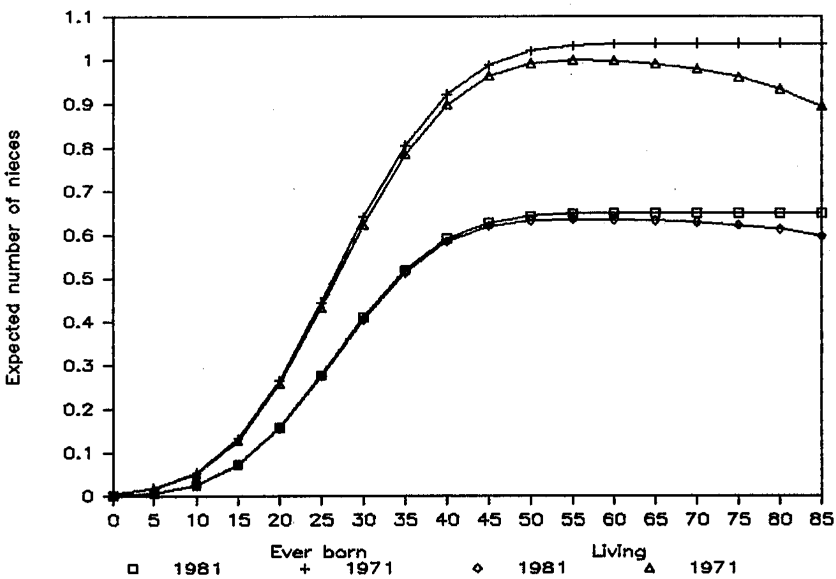

FIGURE 6A. TOTAL NIECES BORN AND LIVING
CANADA 1981 AND 1971

Aunts

Table 7 and Figure 7 show the number of aunts through older and younger sisters of the mother of Ego. One's mother cannot have any further older sisters, so one cannot have any further aunts in this category. One's mother could have further younger sisters, but not many do. At age 0 the expected number of aunts for 1971 in total is 1.029, while by age 20 it is 1.033, a difference of only 0.004. For 1981 the difference is even smaller – 0.001. The ratio for 1981 of aunts through younger sisters of the mother to aunts through older sisters is the same 0.954 that appears for sisters and nieces. The Goldman (1978) device can be applied equally to information on sisters, nieces and aunts.

Aunts born and aunts living are shown in Figure 7A. The aunts born are nearly horizontal lines, whereas the aunts alive diminish fairly rapidly, starting a certain distance below the aunts born. Once again the curve for 1971

TABLE 7. NUMBER OF AUNTS EVER BORN

AGE	AUNTS THROUGH OLDER SISTERS 1981	AUNTS THROUGH OLDER SISTERS 1971	AUNTS THROUGH YOUNGER SISTERS 1981	AUNTS THROUGH YOUNGER SISTERS 1971	TOTAL AUNTS 1981	TOTAL AUNTS 1971
0	0.415	0.516	0.395	0.513	0.810	1.029
20	0.415	0.516	0.396	0.517	0.811	1.033
40	0.415	0.516	0.396	0.517	0.811	1.033
60	0.415	0.516	0.396	0.517	0.811	1.033
80	0.415	0.516	0.396	0.517	0.811	1.033

TABLE 8. NUMBER OF AUNTS ALIVE

AGE	AUNTS THROUGH OLDER SISTERS 1981	AUNTS THROUGH OLDER SISTERS 1971	AUNTS THROUGH YOUNGER SISTERS 1981	AUNTS THROUGH YOUNGER SISTERS 1971	TOTAL AUNTS 1981	TOTAL AUNTS 1971
0	0.407	0.497	0.389	0.500	0.796	0.997
20	0.394	0.466	0.386	0.493	0.780	0.959
40	0.317	0.333	0.363	0.443	0.680	0.776
60	0.073	0.072	0.232	0.258	0.305	0.330
80	0.000	0.000	0.015	0.023	0.015	0.023

drops faster than that for 1981, and the two come relatively close after about age 40 of the person – beyond that age on this model the 1981 rates show about the same number of living aunts as the 1971, though the 1971 are initially much higher.

Cousins

The number of cousins through an older sister of the mother bears for 1981 the ratio of 1.048 to the number of cousins through a younger sister, the same ratio as for sisters and nieces. Once again, we are assuming averages without taking account of individual variation, and so the number of children of the mother's siblings is proportional to the number of those siblings.

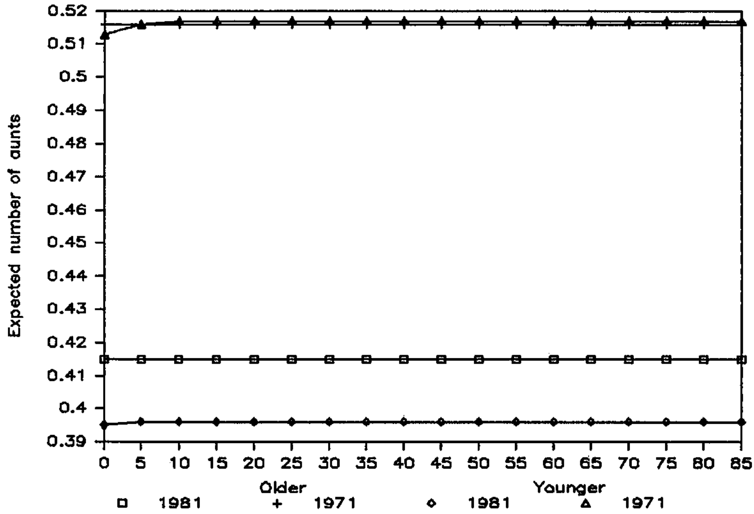

FIGURE 7. NUMBER OF AUNTS BORN, CANADA 1981 AND 1971

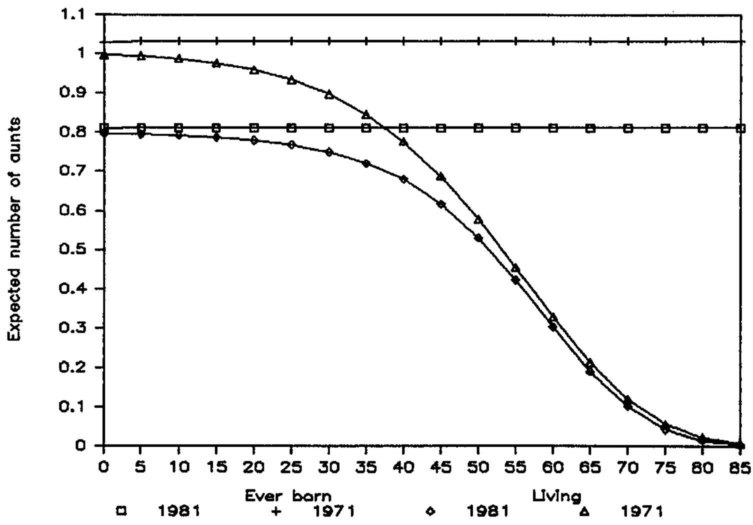

FIGURE 7A. TOTAL AUNTS BORN AND LIVING
CANADA 1981 AND 1971

TABLE 9. NUMBER OF COUSINS EVER BORN

	COUSINS THROUGH OLDER SISTERS		COUSINS THROUGH YOUNGER SISTERS		TOTAL COUSINS	
AGE	1981	1971	1981	1971	1981	1971
0	0.252	0.389	0.083	0.130	0.335	0.519
20	0.332	0.518	0.294	0.466	0.626	0.984
40	0.332	0.519	0.317	0.520	0.649	1.039
60	0.332	0.519	0.317	0.520	0.649	1.039
80	0.332	0.519	0.317	0.520	0.649	1.039

TABLE 10. NUMBER OF COUSINS ALIVE

	COUSINS THROUGH OLDER SISTERS		COUSINS THROUGH YOUNGER SISTERS		TOTAL COUSINS	
AGE	1981	1971	1981	1971	1981	1971
0	0.250	0.380	0.082	0.126	0.331	0.506
20	0.326	0.502	0.291	0.455	0.617	0.957
40	0.320	0.483	0.310	0.500	0.630	0.983
60	0.282	0.392	0.299	0.468	0.581	0.860
80	0.133	0.167	0.234	0.337	0.367	0.503

Figure 8 shows the number of cousins born and living. One does not acquire further cousins after about age 25, and the cousins one has are reduced by mortality. The ratio of 1971 to 1981 is 1.601, the same as it was for nieces. The ratio is not very different from that for grandchildren, which is 1.578, and only slightly less than the square of the ratio of daughters born, 1.623.

Comparative Statics Versus the Process of Change

The results obtained here are those of comparative statics, which is to say that they tell what would happen if the given regime of fertility and mortality (i.e. that of Canada 1971 or 1981) were to continue indefinitely. They are exactly true on the stated assumptions, which include rates of birth and death fixed at the specified levels.

It would be of interest to trace the changing pattern of kinship, but that is beyond the scope of this article. For one thing, the mathematical problem of tracing the change theoretically, whether or not it is possible, is beyond the capacity of this writer. The trajectory of cousins, for instance, is determined once the trajectory of births and deaths is known; so there must be some way of finding it, but this is a problem not yet tackled.

Secondly, this dynamic problem would give a result dependent upon the particular trajectory that one chose for the future births and deaths. That makes the interpretation of its results more intricate. Instead of being able to say, as we do here, that such and such birth and death rates imply such and such kinship patterns, one would have to say that if the future births move along a certain given path, then the kinship will move along a certain path; and if births move in some other way kinship will do something else. The comparative statics results have at least the advantage of simplicity.

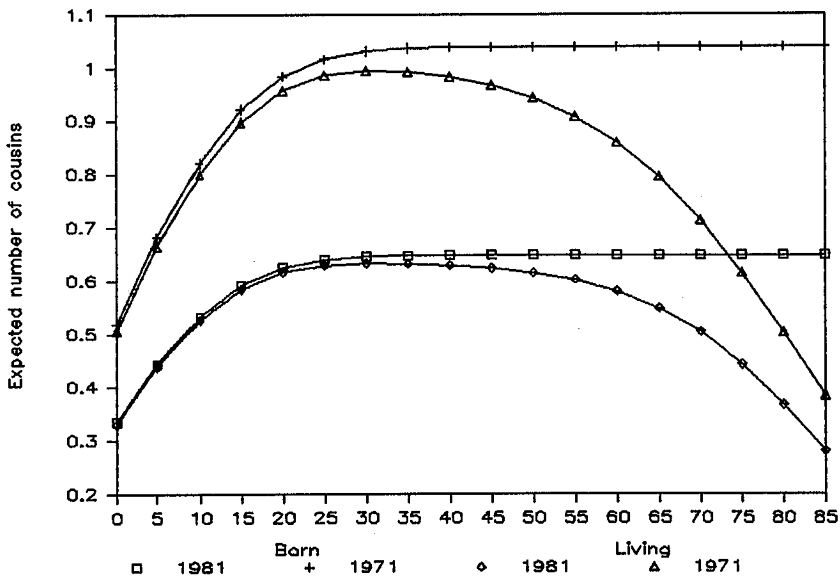

FIGURE 8. COUSINS BORN AND LIVING
CANADA 1981 AND 1971

Survey Information

A further distinction is that between the results obtained here and those of an actual survey. One might enumerate a probability sample of the population, asking each individual whether their mother is alive, how many first cousins the person ever had, how many are alive, and all of the other relations reported in this article. This would give quite different results from those specified here, in that it would reflect the past trajectory of birth and death rates. If fertility has been falling, the survey would show larger numbers of kin than this article's method based on present rates.

If one has the resources to carry out such a survey, one has to worry about the accuracy of statements, especially for births of more distant kin. If sufficient accuracy could be attained, the survey results would establish, for instance, the market for greeting cards addressed to cousins. The number of children, as reported in a survey, would be the right figure to use in planning housing. If one were to fix a certain responsibility for parents on their children, the actual survey number of parents having living children would provide useful estimates for the future. But no survey could tell what kin are implied by what birth and death rates.

In short, the material resources needed to interview a usable sample of the population for this specific purpose would be considerable, and the results could not be trusted for distant kin (and certainly not for deceased kin). The dynamic problem, on the other hand, is too difficult mathematically and would anyway give complex results. The comparative statics reported here are not equivalent to these more expensive or more difficult approaches, providing the kin numbers implied by a given regime of mortality and fertility when all else is held constant.

The Sexes

The results in this article have been expressed in terms of females only. An additional set of results could be produced for males, with one qualification only: the ages of males would be measured from conception rather than birth. We know that a mother was alive at the birth of her child; all we know about a father is that he was alive nine months previously. The age incidence of reproduction for males is slightly different to that for females, so the age of childbearing input would have to be altered; and male longevity is less than that of females for most populations, so that too would be somewhat different. But none of these would greatly affect the numerical results, and insofar as they do not, the results of this article are good approximations to what one would obtain in the corresponding calculations for brothers, uncles, nephews and male cousins. To a reasonable approximation, the kin of both sexes combined are given by doubling the numbers in our tables.

The Birth Pattern for Individual Families

One assumption of this article offers special difficulties. We have said that the formulas incorporate the age-specific rates of birth and death as inputs, their outputs being the number of kin. But insofar as they use the infinitesimal calculus, they do not suppose that children are born as units, with occasional multiple births, but rather that each woman gives birth in each interval of age dx to $m(x)dx$ of a child. There is no corresponding difficulty for mortality, so those formulas that depend mostly on mortality, for instance, number of living ancestors, are not affected. Formulas that depend on fertility, but only on its average amount, such as the expected number of children, grandchildren, and so on ever born or now alive, are not affected either.

Those formulas that are dependent on the distribution of children among families are open to suspicion. For instance, the number of sisters that the average girl has could in principle vary greatly for a given level of overall average fertility. This average might be one girl per family, but it makes a difference to the number of sisters whether each couple has one girl, or one-sixth of couples have six girls and the remaining five-sixths have none. In the one case, the average number of sisters is zero, in the other case it is five; and both are consistent with the given average of one. The point is discussed in Goodman, Keyfitz and Pullum (1975).

Simulation experiments can check the effect of this and some of these have shown that the unrealistic assumption does not seriously distort the results (Le Bras, 1983). Further checking along this line is proceeding.

Mathematical Expressions Required

All the formulas used, are contained in Goodman, Keyfitz and Pullum (1974) and given at the end of this article. Here, I confine myself to a non-technical form of the argument behind two of the formulas, the logic of which can be taken to represent the others.

The expected number of granddaughters ever born of a woman aged a is

$$\int_{\alpha}^{a} \left[\int_{\alpha}^{a-x} l(y)m(y)dy \right] m(x)dx$$

where $l(y)$ is the probability of surviving from birth to age y and $m(y)dy$ the chance of a birth to a woman aged y before she attains y + dy. To appreciate the formula, note that the age-specific rate of birth being $m(x)$, the expected number of children that a woman would bear in the small interval dx of age is $m(x)dx$. The quantity x is here taken over the range of ages from α, the first fertile age, to a, the present age of the woman.

Let us designate the woman aged a with A, her daughter with X, and her granddaughter with Y. Since at the end of the range of ages (α to a) the woman A is alive, we have no need for a factor for mortality in the outer integral. The inner integral calculates the expected number of children born to the child X who were born by age x of the initial woman A. We do not know that these children will live, and hence need $l(y)$ to provide for their survivorship, then to multiply by $m(y)dy$ for the number of children Y that they will have at age y to y + dy if they do live until then. And this has to be integrated from the earliest fertile age α to the age at which the daughter X is now, when the initial woman A is aged a (i.e., from α to a - x).

So much for granddaughters born. Since not all survive, to ascertain the number who do survive, a further factor $l(a\text{-}x\text{-}y)$ must be entered in the inner integrand. To find the number of great- granddaughters one needs a further integral, within the integral for daughters. The calculations of this paper go as far as great-great-granddaughters, and so involve multiple integrals nested at four levels.

For sisters, the division according to older and younger is unavoidable, since we know that the mother was alive up to the time of the birth of the child X, but do not know how long she will live after that. Like many other kinship formulas, those for sisters are in two steps, the first being conditional on the woman aged a having been born when her mother was aged x, and the second step being the removal of the condition by averaging. We assume that the girl A aged a is chosen at random from the population. Under the condition that her mother X was aged x when she was born, the number of older sisters is nothing more complicated than the integral of $m(y)$ from α to x. Multiplying this by the age distribution of mothers, which in the stable case is $e^{-rx}l(x)m(x)dx$, where r is Lotka's (1931) intrinsic rate of natural increase, and then integrating over x, gives the unconditional number of older sisters born. For older live sisters we need only incorporate a survival factor $l(a+$x-y) within the integral.

For younger sisters, a factor for the survivorship of the mother is required:

$$\int_{\alpha}^{\beta} [\int_{o}^{a} (l(x+y)/l(x))m(m+y)dy] \, e^{-rx}l(x)m(x)dx.$$

Using essentially the same techniques, the Goodman, Keyfitz and Pullum (1974) paper presented formulas for the 26 kin relations that are shown in this paper, and Pullum programmed these in FORTRAN. With some modification of its output, the Pullum program provided the numbers given in this paper.

Appendix on Data

The table below provides all the information used as input to the formulas. The formulas being in continuous terms, and the data in five-year age groups, we needed finite approximations. The construction of such approximations was central to the programming task undertaken by Tom Pullum.

CANADA BIRTH RATES AND SURVIVORSHIPS

AGE	BIRTH RATES 1981	SURVIVORSHIPS
0-4	0.0	4.95566
5-9	0.0	4.94668
10-14	0.00029	4.94120
15-19	0.02569	4.94120
20-24	0.09468	4.93283
25-29	0.12423	4.92141
30-34	0.06665	4.90422
35-39	0.01901	4.89513
40-44	0.00314	4.87557
45-49	0.00019	4.84430
50-54	0.0	4.79499
55-59	0.0	4.71735
60-64	0.0	4.59787
65-69	0.0	4.42037
70-74	0.0	4.15717
75-79	0.0	3.77110
80-84	0.0	2.46862
85-89	0.0	2.68940

TOTAL GRR = 0.81251 e_0 = 80.77507
INTRINSIC RATE OF NATURAL INCREASE = -0.0082

CANADA BIRTH RATES AND SURVIVORSHIPS (continued)

AGE	BIRTH RATES 1971	SURVIVORSHIPS
0-4	0.0	4.81940
5-9	0.0	4.90482
10-14	0.00026	4.89678
15-19	0.03865	4.88610
20-24	0.13118	4.87219
25-29	0.13891	4.85776
30-34	0.07567	4.83942
35-39	0.03292	4.81303
40-44	0.00921	4.77290
45-49	0.00060	4.71275
50-54	0.0	4.62146
55-59	0.0	4.48418
60-64	0.0	4.28396
65-69	0.0	3.98996
70-74	0.0	3.55712
75-79	0.0	2.93944
80-84	0.0	2.12984
85-89	0.0	1.97296

TOTAL GRR = 1.03576 e_0 = 76.35408
INTRINSIC RATE OF NATURAL INCREASE = 0.0002

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The formulas are derived in Goodman, Keyfitz, and Pullum (1974), and are expressed in a notation that follows the usual conventions of demography:

l(x)	probability of surviving from zero to age x
m(x)dx	probability that a woman aged x will have a child before age $x+dx$
alpha	beginning of reproductive interval
beta	end of reproductive interval
W(x)	$e(-rx)l(x)m(x)$ age distribution of childbearing in stable population
r	intrinsic rate of natural increase, such that the integral of e(-rx)l(x)m(x) over the range of childbearing is equal to unity.

FOR A WOMAN NOW AGE A, THE EXPECTED NUMBER OF DAUGHTERS EVER BORN

$$\int_{\alpha}^{a} m_x \, dx. \qquad (1.1a)$$

DAUGHTERS STILL ALIVE

$$\int_{\alpha}^{a} l_{a-x} m_x \, dx. \qquad (1.1b)$$

GRANDDAUGHTERS EVER BORN

$$\int_{\alpha}^{a} \left[\int_{\alpha}^{a-x} l_y m_y \, dy \right] m_x \, dx. \qquad (1.2a)$$

GRANDDAUGHTERS STILL ALIVE

$$\int_{\alpha}^{a} \left[\int_{\alpha}^{a-x} l_y m_y l_{a-x-y} \, dy \right] m_x \, dx. \qquad (1.2b)$$

GREAT GRANDDAUGHTERS EVER BORN

$$\int_{\alpha}^{a} \left[\int_{\alpha}^{a-x} \left\{ \int_{\alpha}^{a-x-y} l_z m_z \, dz \right\} l_y m_y \, dy \right] m_x \, dx. \qquad (1.3a)$$

THE PROBABILITY HER MOTHER IS STILL ALIVE

$$M_1(a) = \int_\alpha^\beta (l_{a+x}/l_x)W(x)dx = \int_\alpha^\beta l_{a+x}m_x e^{-rx}dx.$$

GRANDMOTHER IS STILL ALIVE

$$M_2(a) = \int_\alpha^\beta M_1(a+x)W(x)dx, \tag{2.2}$$

GREAT GRANDMOTHER IS STILL ALIVE

$$M_3(a) = \int_\alpha^\beta M_2(a+x)W(x)dx, \tag{2.3}$$

GREAT GREAT GRANDMOTHER IS STILL ALIVE

$$M_4(a) = \int_\alpha^\beta M_3(a+x)W(x)dx. \tag{2.4}$$

FOR A WOMAN NOW AGE A, THE EXPECTED NUMBER OF OLDER SISTERS EVER BORN

$$\int_\alpha^\beta \left[\int_\alpha^x m_y dy\right]W(x)dx, \tag{3.1a}$$

YOUNGER SISTERS EVER BORN

$$\int_\alpha^\beta \left[\int_0^a (l_{x+y}/l_x)m_{x+y}dy\right]W(x)dx. \tag{3.2a}$$

NUMBER OF OLDER SISTERS STILL ALIVE

$$\int_\alpha^\beta \left[\int_\alpha^x m_y l_{a+x-y}dy\right]W(x)dx. \tag{3.1b}$$

YOUNGER SISTERS STILL ALIVE

$$\int_\alpha^\beta \left[\int_0^a (l_{x+y}/l_x)m_{x+y}l_{a-y}dy\right]W(x)dx. \tag{3.2b}$$

FOR A WOMAN NOW AGE A, THE EXPECTED NIECES EVER BORN VIA OLDER SISTERS

$$\int_{\alpha}^{\beta} \left[\int_{\alpha}^{x} \left\{ \int_{\alpha}^{a+x-y} l_z m_z \, dz \right\} m_y \, dy \right] W(x) dx. \tag{4.1a}$$

VIA YOUNGER SISTERS

$$\int_{\alpha}^{\beta} \left[\int_{0}^{a} \left\{ \int_{\alpha}^{a-y} l_z m_z \, dz \right\} (l_{x+y} / l_x) m_{x+y} \, dy \right] W(x) dx. \tag{4.2a}$$

NUMBER OF NIECES VIA OLDER SISTERS STILL ALIVE

$$\int_{\alpha}^{\beta} \left[\int_{\alpha}^{x} \left\{ \int_{\alpha}^{a+x-y} l_z m_z l_{a+x-y} \, dz \right\} m_y \, dy \right] W(x) dx. \tag{4.1b}$$

FOR A WOMAN NOW AGE A, THE EXPECTED AUNTS EVER BORN BEFORE THE MOTHER

$$\int_{\alpha}^{\beta} \left[\int_{\alpha}^{\beta} \left\{ \int_{\alpha}^{y} m_z \, dz \right\} W(y) dy \right] W(x) dx. \tag{5.1a}$$

AFTER THE MOTHER

$$\int_{\alpha}^{\beta} \left[\int_{\alpha}^{\beta} \left\{ \int_{0}^{a+x} (l_{y+w} / l_y) m_{y+w} \, dw \right\} W(y) dy \right] W(x) dx. \tag{5.2a}$$

FOR A WOMAN NOW AGE A, THE EXPECTED COUSINS BORN VIA MOTHER'S OLDER SISTER

$$\int_{\alpha}^{\beta} \left[\int_{\alpha}^{\beta} \left\{ \int_{\alpha}^{y} \left[\int_{\alpha}^{a+x+y-z} l_w m_w \, dw \right] m_z \, dz \right\} W(y) dy \right] W(x) dx. \tag{6.1a}$$

VIA YOUNGER SISTERS

$$\int_{\alpha}^{\beta} \left[\int_{\alpha}^{\beta} \left\{ \int_{y}^{a+x+y} \left[\int_{\alpha}^{a+x+y-z} l_w m_w \, dw \right] \frac{l_z}{l_y} m_z \, dz \right\} W(y) dy \right] W(x) dx, \tag{6.2a}$$

NUMBER OF COUSINS STILL ALIVE VIA MOTHER'S OLDER SISTERS

$$\int_{\alpha}^{\beta}\left[\int_{\alpha}^{\beta}\left\{\int_{\alpha}^{y}\left[\int_{\alpha}^{a+x+y-z} l_w m_w l_{a+x+y-z-w}\,dw\right] m_z\,dz\right\} W(y)\,dy\right] W(x)\,dx \tag{6.1b}$$

VIA YOUNGER SISTERS

$$\int_{\alpha}^{\beta}\left[\int_{\alpha}^{\beta}\left\{\int_{y}^{a+x+y}\left[\int_{\alpha}^{a+x+y-z} l_w m_w l_{a+x+y-z-w}\,dw\right]\frac{l_z}{l_y} m_z\,dz\right\} W(y)\,dy\right] W(x)\,dx \tag{6.2b}$$