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ON ESTIMATING THE MEAN BIRTH INTERVAL CHARACTERISTIC OF WOMEN

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Résumé – Nous suggérons une alternative à la méthode de Wolfers (1968) pour obtenir l'intervalle généstique moyen à partir des données sur les intervalles entre les naissances pour les enfants. La méthode est appliquée aux donnés d'une enquête à Varanasi de 1969-70 et aux données de Wolfers. Nous trouvons que la nouvelle méthode est relativement simple et sans problèmes.

Abstract – An alternative procedure to the method suggested by Wolfers (1968) for obtaining the mean birth interval characteristic of women from the data on birth interval (births) has been suggested. It has been shown that the Wolfers procedure involves some methodological problems. The present procedure, having been applied to an observed set of data taken from the Varanasi Survey 1969-70 and to the data cited by Wolfers, is found to be relatively simple, and free from those methodological problems.

Key Words – **birth interval (births), birth interval (women), asymptotic fertility rate, augmented frequencies**

Introduction

When discussing the problem of selection bias in the birth interval analysis, Wolfers (1968) distinguished between average birth interval based on all births that occurred in a given period and the birth interval characteristic of women in a population and termed these two as birth interval (births) and birth interval (women), respectively. He has given a procedure to estimate the mean birth interval (women) in a population from the data on birth interval (births). However, the procedure involved some methodological problems: He derived the theoretical expression for the variance, V, of birth interval (women), whereas at the time of application the variance of birth interval (births) used may have been quite different. Besides, the expression for the within group variance, $V_2 = (A-W)W$ (Wolfers, 1968, equation 1, p. 257), seems confusing and may be invalid for some situations. Also, the estimate of the mean non-susceptible period, C, knowledge of which is required in the procedure, was obtained heuristically, without using any objective criterion.

In this note, we have discussed an alternative procedure for the above situation, which is not only free from the methodological problems but also simple in its application. A brief discussion of the methodological problems of the Wolfers procedure has been given through an illustrative example in the appendix.

The Alternative Procedure and Its Application

We derive an alternative procedure for estimating the mean birth interval (women) from the data on birth interval (births) following a method discussed by Singh *et. al.*, (1979) for the adjustment of selection bias in the postpartum amenorrhea (PPA).

Let there be a population of married women heterogeneous with respect to both the period of non-susceptibility g and fecundability ϕ. Let the joint and marginal distribution of ϕ and g be denoted by $f(\phi,g)$, $f_1(\phi)$ and $f_2(g)$ respectively. Let $\psi_{g,\phi} = \dfrac{\phi}{1+g\phi}$ be the asymptotic fertility rate for the given ϕ as well as g, and let ψ_g be the asymptotic fertility rate for the given g alone, and let ψ_ϕ be the one when ϕ alone is given. Therefore, as discussed in Henry (1972), ψ is considered the asymptotic fertility rate in the population. In a population where the women giving birth in a small interval of time are included in a study, the distribution of ϕ and g will be

$$f_3(\phi) = \frac{\psi_\phi}{\psi} f_2(\phi) \text{ and } f_4(g) = \frac{\psi_g}{\psi} f_2(g) \tag{1}$$

and their joint density is given by

$$f_5(\phi,g) = \frac{\psi_{\phi,g}}{\psi} f(\phi,g) \tag{2}$$

It can easily be seen that $f_3(\phi)$ and $f_4(g)$ are quite different respectively from $f_1(\phi)$ and $f_2(g)$. Now let X denote the length of the birth interval of the women giving birth in a small interval of time, then

$$E(X) = \iint \frac{1+g\,\phi}{\phi} f_5(\phi,g)\, d\phi dg = \frac{1}{\psi} = M \text{ (say)} \tag{3}$$

and

$$E(X^2) = \iint \frac{(1+g\phi)^2}{\phi^2} \cdot \frac{\phi}{1+g\phi} \cdot \frac{1}{\psi} f(\phi,g)\, d\phi dg$$

$$= W.M. + \iint \frac{M}{\phi(1+g\phi)} f(\phi,g)\, d\phi dg$$

where

$W = \iint \frac{1+g\phi}{\phi} f(\phi,g)\, d\phi dg$ is the mean birth interval (women).

Thus,

$$\frac{E(X^2)}{M} = W + \iint \frac{1}{\phi\,(1+g\phi)} f(\phi,g)\, d\phi dg \tag{4}$$

Proceeding as discussed in Singh *et al.*, (1979), and denoting $\frac{E(X^2)}{M} = A$ (as given in Wolfers, 1968), it is obtained that

$$A = 2\,W - \bar{g} - \bar{g}'\,(1 - \frac{\bar{g}'}{M}) \tag{5}$$

where $\bar{g} = \int g f_2(g)\, dg$ and $\bar{g}' = \int g f_4(g)\, dg$

It may be pointed out that the above expression is obtained by assuming $\bar{g}'$ to be constant with respect to ϕ. Thus, the value of W can be estimated from equation (5) with the knowledge of the values of A, M, $\bar{g}$ *and* $\bar{g}'$. In a study where data on birth intervals and the length of PPA are available from the women giving birth in a fixed period of time, the values of A, M and $\bar{g}'$ are known and the value of $\bar{g}$ can be estimated from the expression $\bar{g}' = \bar{g} - \frac{\sigma'^2}{M}$, where σ'^2 is the variance of the duration of PPA for women giving birth in a small interval of time. For this purpose, the distribution of the length of the interval between the last and the last-but-one births for women giving birth during the last two years from the reference date of the survey (Varanasi Survey, October, 1969, details of which are given in Singh *et al.*, 1970) was considered, and the values of A and M are found to be 42.24 and 35.49 months, respectively. The values of $\bar{g}$ and $\bar{g}'$ were taken from Table 2 of Singh *et al.*, (1979). On the basis of these values, the estimate of W has been obtained as 36.16 months. When all females with marriage duration of more than five years were considered, and when their last birth interval was measured, it was found that the average birth interval came out to be 36.55 months. This may be considred as an observed value of W. Thus, it is noticed that the two estimates of W, i.e., one obtained by using the formula given in equation (5) and the other by the observed mean of the last closed birth interval, are quite close to each other.

Wolfers (1968) has presented estimates of W in Tables 2 to 7 of his paper, based on the data of birth interval (births). His procedure requires estimates of a ("within group" variance of infecundity), b ("between group" variance of infecundity) and A (the mean of augmented frequencies), M (the mean birth interval (births)) and V (total variance). In our method, we require the estimates of $\bar{g}$ and $\bar{g}'$, apart from the values of A and M. For the application of our method to the data presented in Wolfers (1968), we take $\bar{g}=C$ (in our notation) and $\bar{g}' = \bar{g} - \frac{\sigma'^2}{M}$. Using our notation, we see that $b = \sigma g^2$, where σg^2 is the variance of g corresponding to the probability density function $f_2(g)$. On the basis of some test calculations, it is found that the difference between σ^2/M and σ'^2/M is negligible, and hence we take $\bar{g}' = \bar{g} - \frac{b}{M}$. It should be mentioned here that since we have assumed fecundability and the period of nonsusceptibility to be constant for a woman, the value of a is obviously zero. Taking the estimates of b and C of Wolfers (1968), the estimated values of W, by the present procedure for the data given in Tables 2, 3 and 4 of Wolfers (1968), are presented here in Table 1. For comparison, the estimates of W obtained by Wolfers (1968) are also given in Table 1. A review of Table 1

TABLE 1. A COMPARISON OF THE ESTIMATED VALUES OF *W* GIVEN BY WOLFERS (1968) WITH THE VALUES OBTAINED BY THE PRESENT PROCEDURE

Age in Years	Estimates of W					
	W_{D_2}	W_{S_2}	W_{S_3}	W_{S_3}	W_{D_4}	W_{S_4}
10-19	34.86	34.33	33.25	33.16	32.52	32.49
20-24	34.08	33.85	37.42	36.52	34.55	34.54
25-29	35.42	35.32	33.67	33.70	35.06	35.08
30-34	37.99	38.04	30.92	30.96	35.36	35.42
35-39	35.54	35.59	27.43	27.37	30.53	30.55
40+	31.26	31.09	28.45	28.49	17.99	17.96

Note: W_{D_i} (i = 2, 3, 4) represents the value of W for the ith table in Wolfers (1968) whereas W_{S_i} (i = 2, 3, 4) is the corresponding estimate of W by the procedure proposed in this paper.

shows that although both the procedures are quite different, as the Wolfers procedure is based on the partitioning of the total variance into two components – "within group variance" and "between group variance" – and the present procedure attempts to find the value of A in terms of W and some other fertility parameters, the estimates of W obtained from both the procedures are almost the same. Similar results are also obtained for the data given in Tables 5, 6 and 7 of Wolfers (1968), consequently we do not present them here. Regarding the amount of difference between the values of M and W, it may be noted that it depends upon the nature of the distribution of ϕ and g in the population. For instance, for the probability distribution III and IV (see Appendix Table 1), when the difference between M and W is only about two months, the difference for probability distribution II is about five months. At the same time, empirically we see that the difference between M and W in the Varanasis Survey is only one month, while for the data presented in Wolfers (1968), the differences between M and W are lying in the range of 0.37 to 11.26 months (see Tables 2 to 7 of Wolfers, 1968). This shows that our procedure may have potential for use in getting the refined estimate of W.

References

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APPENDIX

Here we discuss some methodological problems associated with equations (1) and (6) of Wolfers (1968), i.e., $V_2 = (A-W)\, W$ and $V_1 + V_2 = V$, respectively. For this purpose, first we treat time as discrete (unit as month) and take four distributions of ϕ, fecundability, and g, the period of infecundity. Utilizing these four different joint distributions of ϕ and g (given at the bottom of the Appendix Table 1), we compute the values of V_1, V, A, M and W and two values of V_2 from equations (1) and (2) of Wolfers (1968). Further, the estimates of W by the formula of Wolfers (1968) given by

$$W = \frac{2A+2C+1+[(2A+2C+1)^2 - 12(C^2+C+a-b+V)]}{6}$$

are also obtained and presented in the last row of the Appendix Table 1 (see page 8). It is noticed from the table that both equations (1) and (2) of Wolfers (1968) are invalid. Interestingly enough, the estimates of W are found to be quite close to their actual values. This may be perhaps due to cancellation of some errors by combining the equations (1), (2) and (6) for obtaining the estimates of W.

APPENDIX TABLE 1. ILLUSTRATIVE TABULATION FOR EXAMINING THE VALIDITY OF EQUATIONS (1) AND (2) OF WOLFERS (1968)

Quantities	Distributional Combination of ϕ and g			
	I	II	III	IV
V_1	156.85	243.83	39.50	57.79
V_2^*	416.05	515.92	33.95	33.03
V_2^{**}	507.19	680.46	27.73	26.01
V	495.22	624.54	69.79	87.56
A	48.89	52.90	24.68	28.17
M (Actual)	34.56	35.19	21.45	24.65
W (Actual)	37.92	40.00	23.22	26.95
W*** (Estimated)	37.77	39.70	23.40	27.11

* Value of V_2 from equation (1) of Wolfers (1968)

** Value of V_2 from equation (2) of Wolfers (1968)

*** Estimate of W by equation (A)

Distributional Combination of ϕ and g are considered as follows:

							:					
I	Value of ϕ:	.02	.04	.06	.08	.10	:	.1	.2	.4	.2	.1
	Value of g:	10	15	20	25	30	:	.3	.1	.4	.15	.05
II	Value of ϕ:	.02	.04	.06	.08	.10	:	.2	.1	.3	.3	.1
	Value of g:	10	15	20	25	30	:	.3	.1	.4	.15	.05
III	Value of ϕ:	.10	.15	.20	.25	.30	:	.1	.2	.4	.2	.1
	Value of g:	10	15	20	25	30	:	.3	.1	.4	.15	.05
IV	Value of ϕ:	.10	.15	.20	.25	.30	:	.05	.30	.35	.20	.10
	Value of g:	10	15	20	25	.30	:	.125	.250	.1875	.0625	.3750