

A MEASURE OF GROUP BEHAVIOUR, ITS CHARACTERIZATION AND APPLICATIONS IN SOCIO-ANTHROPOLOGICAL STUDIES

A. M. Mathai

McGill University, Montreal, Quebec, Canada
and

P. N. Rathie

Universidade Estadual de Campinas, Campinas, Brazil

Résumé—Dans l'article qui suit nous avons défini une mesure d' "attraction" entre deux distributions statistiques. Nous avons énuméré quelques — unes des propriétés et nous avons donné un théorème de caractérisation à l'aide des postulats. On peut considérer ce théorème de caractérisation comme une définition de la mesure d' "attraction." Nous avons démontré l'utilité de cette mesure en l'appliquant aux données recueillies par une enquête étendue pour la période 1964-67. Un aspect démographique des données, à savoir, le nombre de naissances vivantes chez les femmes, a été utilisé dans notre démonstration. Nous avons mis en lumière la technique pour l'étude des divers groupes religieux en utilisant la mesure d' "attraction". Nous avons démontré comment on peut utiliser cette mesure d' "attraction" dans l'étude de conduite de masse et dans la classification des populations d'après quelques caractéristiques socio-économiques ou anthropologiques.

Abstract—In this article we define a measure of "attraction" between two statistical distributions. A number of its properties are listed and a characterization theorem is given with the help of postulates. This characterization theorem can act as an axiomatic definition for the measure of attraction. The uses of this measure are pointed out by applying the measure to the data collected by an extensive statistical survey over the period 1964-67. One demographic aspect of the data, namely, the number of live births among women, is utilized in the illustration. The technique of studying the "closeness" of various religious groups, as far as the number of live births are concerned, is illustrated and the religious groups are classified by using this measure of "attraction." It is shown how this measure of "attraction" can be used in studying group behaviour and classifying populations according to some socio-economic or anthropological characteristics.

1 Introduction

In socio-anthropological studies group behaviour is a major theme of study. Classifying populations according to the various characteristics is another aspect which has attracted sociologists, anthropologists and statisticians alike. There are various types of measures for studying group behaviour. Of these, statistical and information measures are useful in sociological studies. Concepts such as entropy, inaccuracy, directed-divergence and affinity have found applications in sociological problems. Theoretical aspects as well as a summary of applications of these measures may be found in Mathai and Rathie (1975). Among these the measure of affinity is found to be useful in practical applications. A mathematical foundation as well as some applications of this measure are given in Mathai (1974) and Kaufman and Mathai (1973).

The aim of this article is to introduce a concept of "attraction" between two populations designated by two discrete probability distributions. This measure is seen to be closer to affinity and it has properties similar to the various measures in Information Theory. It is introduced and its properties are studied in section 2 and a mathematical characterization is given in section 3. Its application to practical problems is pointed out in section 4 by applying it to classify data according to some demographic characteristics.

II A Measure of Attraction

In order to define the measure we may start with two discrete probability distributions. Let $P = (p_1, \dots, p_n)$ and $Q = (q_1, \dots, q_n)$ be two discrete distributions. That is, The p s and q s are numbers such that $p_i \geq 0$, $q_i \geq 0$, for $i = 1, \dots, n$, $\sum_{i=1}^n p_i = 1 = \sum_{i=1}^n q_i$. Thus these numbers can represent probabilities corresponding to some events defined in some sample spaces. The measure of affinity between the distributions P and Q , denoted by ρ , is given as

$$\rho = \sum_{i=1}^n (p_i q_i)^{1/2}$$

This measure is extensively used in statistical inference problems, the references of which are available from Mathai (1974). ρ measures some sort of closeness between the populations P and Q . A geometrical interpretation is given as follows. If P and Q are represented as unique points on a hypersphere, then the cosine of the angle between these two vectors is ρ . In this sense it measures the angular separation of the two vectors representing the populations P and Q . Algebraically ρ is a sum of geometric means. But it is well-known that the harmonic mean (H.M.), the geometric mean (G.M.) and the arithmetic mean (A.M.) satisfy the inequalities

$$\text{H.M.} \leq \text{G.M.} \leq \text{A.M.}$$

Hence the harmonic mean can also be used to study the closeness of the populations P and Q . The sum of harmonic means corresponds to

$$A = \sum_{i=1}^n [2p_i q_i / (p_i + q_i)].$$

This quantity A is seen to have properties similar to those of affinity as well as those of the various measures in Information Theory. Hence we define A as the measure of "attraction" between P and Q . This is a function of the probabilities $p_1, \dots, p_n$ and $q_1, \dots, q_n$ and hence for convenience we denote it by $A_n(p_1, \dots, p_n; q_1, \dots, q_n)$. Thus we have, for $p_i \geq 0$, $\sum_{i=1}^n p_i = 1$, $q_i \geq 0$, $\sum_{i=1}^n q_i = 1$, $(p_i, q_i) \neq (0, 0)$ or $(1, 1)$

$$A_n(p_1, \dots, p_n; q_1, \dots, q_n) = \sum_{i=1}^n [2p_i q_i / (p_i + q_i)], \quad n \geq 2$$

For example, for $n = 2$,

$$A_2(p, 1-p; q, 1-q) = 2pq/(p+q) + 2(1-p)(1-q)/(2-p-q)$$

for $p, q \in [0, 1]$, $(p, q) \neq (0, 0)$ or $(1, 1)$.

2.1 Properties of "Attraction" Between P and Q

Here we list the various properties which are similar to those enjoyed by information measures. These properties can be used to give characterization theorems or axiomatic definitions. These will also be helpful for the experimenter looking for a suitable measure for a problem under consideration. He can examine the experimental conditions and select the measure having desirable properties. It may be easily seen that the following are desirable properties for any measure of closeness between two populations. For example, if the two populations are one and the same, that is when $P = Q$, we would like to have maximum "attraction" between them. Since non-mathematical explanation is too long, we list the properties without much explanation.

1. Normalization. When $P = Q$, $A_n(p_1, \dots, p_n; p_1, \dots, p_n) = 1$.
2. Symmetry. $A_n(p_1, \dots, p_n; q_1, \dots, q_n) = A_n(p_{\alpha_1}, \dots, p_{\alpha_n}; q_{\alpha_1}, \dots, q_{\alpha_n})$ where $(\alpha_1, \dots, \alpha_n)$ is an arbitrary permutation of $(1, \dots, n)$.
Also $A_n(p_1, \dots, p_n; q_1, \dots, q_n) = A_n(q_1, \dots, q_n; p_1, \dots, p_n)$.
3. Recursivity. $A_n(p_1, \dots, p_n; q_1, \dots, q_n) = A_{n-1}(p_1 + p_2, p_3, \dots, p_n; q_1 + q_2, q_3, \dots, q_n) - (p_1 q_2 - p_2 q_1)^2 [p_1(q_1 + q_2) + q_1(p_1 + p_2)] [p_2(q_1 + q_2) + q_2(p_1 + p_2)] / (p_1 + p_2 + q_1 + q_2)(p_1 + q_1)(p_2 + q_2)[p_1 p_2 (q_1 + q_2)^2 + q_1 q_2 (p_1 + p_2)^2]$

- $\times A_2(\frac{p_1}{p_1+p_2}, \frac{p_2}{p_1+p_2}; \frac{q_1}{q_1+q_2}, \frac{q_2}{q_1+q_2})$ for $p_1+p_2, q_1+q_2, p_1+q_1, p_2+q_2 > 0$
4. Non-negativity. $0 \leq A_n(p_1, \dots, p_n; q_1, \dots, q_n) \leq 1$
5. Strong Additivity. For $\sum_{i=1}^n p_i = 1, \sum_{j=1}^m p_i^{(j)} = 1, j = 1, \dots, n,$
 $\sum_{i=1}^m Q_i^{(j)} = 1, j = 1, \dots, n,$
 $A_{m_1+\dots+m_n}(p_1 P_1^{(1)}, \dots, p_1 P_{m_1}^{(1)}, p_2 P_1^{(2)}, \dots, p_2 P_{m_2}^{(2)}, \dots, p_n P_1^{(n)}, \dots, p_n P_{m_n}^{(n)};$
 $p_1 Q_1^{(1)}, \dots, p_1 Q_{m_1}^{(1)}, p_2 Q_1^{(2)}, \dots, p_2 Q_{m_2}^{(2)}, \dots, p_n Q_1^{(n)}, \dots, p_n Q_{m_n}^{(n)})$
 $= \sum_{j=1}^n p_j A_{m_j}(P_1^{(j)}, \dots, P_{m_j}^{(j)}; Q_1^{(j)}, \dots, Q_{m_j}^{(j)})$
- This, when $n = 2$, is the following: For $0 \leq p \leq 1, \sum p_i = 1,$
 $A_{m+n}(pp_1, \dots, pp_n, (1-p)r_1, \dots, (1-p)r_m; pq_1, \dots, pq_n, (1-p)s_1, \dots, (1-p)s_m)$
 $= p A_n(p_1, \dots, p_n; q_1, \dots, q_n) + (1-p) A_m(r_1, \dots, r_m; s_1, \dots, s_m)$
6. Structure. $A_n(p_1, \dots, p_n; q_1, \dots, q_n) = \sum_{i=1}^n f(p_i, q_i)$
where $f(p_i, q_i) = 2p_i q_i / (p_i + q_i)$
7. For $\sum_{i=1}^n p_i = 1, \sum_{i=1}^m q_i = 1, \sum_{i=1}^n r_i = 1,$
 $A_{n+m-1}(p, \dots, p_{n-1}, p_n q_1, \dots, p_n q_m; r_1, \dots, r_{n-1}, r_n q_1, \dots, r_n q_m)$
 $= A_n(p_1, \dots, p_n; r_1, \dots, r_n)$

III An Axiomatic Definition

It is customary to give axiomatic definitions to a new measure whenever it is introduced. This will put it on mathematical foundations. These definitions are given in the form of results which establish that an arbitrary function satisfying a set of given postulates is uniquely determined as the measure under consideration. These postulates are usually a subset of the desirable properties of the measure. Different subsets of properties may be used to give characterizations to it. In this section we give one characterization by using a set of three postulates. Other characterizations may be given using other sets of postulates as well. This characterization is given here in the form of a theorem.

Theorem. Let $P = (p_1, \dots, p_n), Q = (q_1, \dots, q_n), p_i \geq 0, q_i \geq 0, \sum_{i=1}^n p_i = 1 = \sum_{i=1}^n q_i$ be two discrete distributions. Let $A_n(p_1, \dots, p_n; q_1, \dots, q_n)$ for $n \geq 2$ be an arbitrary function of the p s and q s. Let $A_n(p_1, \dots, p_n; q_1, \dots, q_n)$ satisfy the following postulates

P1: Recursivity (Property 3 of section 2)

P2: Symmetry (Property 2 of section 2 for $n = 3$)

P3: Normalization $A_2(0, 1; 1/2, 1/2) = 2/3$.

Then A_n is uniquely determined as $A_n = \sum_{i=1}^n [2p_i q_i / (p_i + q_i)]$.

Proof. Taking the postulate P2 in the form

$$A_3(x, 1-x-u, u; y, 1-y-v, v) = A_3(u, 1-x-u, x; v, 1-y-v, y)$$

and using the postulate P1 we get

$$(1) f(u, v) - [\|x(1-v) - y(1-u)\|^2 \|x(1-v) + y(1-u)\| \\ \| (1-x-u)(1-v) + (1-y-v)(1-u) \| \\ / (2-u-v)(x+y)(2-x-y-u-v) \| x(1-x-u)(1-v)^2 + \\ y(1-y-v)(1-u)^2 \|] \times f(x/(1-u), y/(1-v)) \\ = f(x, y) - [\|u(1-y) - v(1-x)\|^2 \|u(1-y) \\ + v(1-x)\| \| (1-x-u)(1-y) + (1-y-v)(1-x) \| / \\ (2-x-y)(u+v)(2-x-y-u-v) \times \|u(1-x-u)(1-y)^2 \\ + v(1-y-v)(1-x)^2 \|] f(u/(1-x), v/(1-y)),$$

for $x, y, u, v \in [0, 1], x+u, y+v \in [0, 1], (x, y) \neq (0, 0), (0, 1-v), (1-u, 0), (1-u, 1-v), (u, v) \neq (0, 0), (0, 1-y), (1-x, 0)$ where $f(x, y)$ stands for $A_2(x, 1-x; y, 1-y), x, y \in [0, 1], (x, y) \neq (0, 0) \text{ or } (1, 1)$.

The postulate P_2 may be rewritten as

$$A_3(p_1, p_2, p_3 : q_1, q_2, q_3) = A_3(p_2, p_1, p_3 : q_2, q_1, q_3)$$

This on utilizing the postulate P_1 gives

$$(2) f(x, y) = f(1 - x, 1 - y) \text{ for all } x, y \in [0, 1]$$

The equation (1) with $x = 0, y = 1/2, v = 0$, on using postulate P_3 yields

$$(3) f(u, 0) = 2(1 - u) / (2 - u), u \in]0, 1[$$

Hence (2) and (3) imply

$$(4) f(1 - u, 1) = 2(1 - u) / (2 - u), u \in]0, 1[$$

Now substituting $x = 0, v = 0$ in (1) and utilizing (3), we have from (1),

$$(5) f(0, y) = 2(1 - y) / (2 - y), y \in]0, 1[$$

Thus (5) and (2) give

$$(6) f(1, 1 - y) = 2(1 - y) / (2 - y), y \in]0, 1[$$

Replacing x by y and u by v in (1) we have $f(u, u) = f(x, x), x, u \in]0, 1[$

with $x + u \in]0, 1[$ giving,

$$(7) f(x, x) = c \text{ where } c \text{ is a constant.}$$

For $x = 1 - u = y$, (1) reduces to

$$(8) f(u, v) - (1/2)(u - v)f(1, (1 - u) / (1 - v)) = f(1 - u, 1 - v) - (1/2)[u(u - v)/v]f(1, v/u),$$

which on utilizing (7), (6) and (2) give

$$(9) f(u, v) = c - (u - v)^2 / [(u + v)(2 - u - v)], \text{ for } u, v \in]0, 1[$$

Substituting the expression for $f(u, v), u, v \in]0, 1[$ from (9) throughout in (1) and simplifying we have

$$(10) c = 1.$$

Hence (9) and (10) give

$$(11) f(u, v) = 2(u + v - u^2 - v^2) / [(u + v)(2 - u - v)], u, v \in]0, 1[$$

Hence (11), (6), (5), (4), (3) and (2) give $f(x, y)$ for $x, y \in [0, 1]$. That is, $A_2(x, 1 - x : y, 1 - y)$ is completely determined. Hence by using the postulate P_1 successively we arrive at the result by induction. This completes the proof.

IV Application to a Socio-Anthropological Problem

The data for this study were collected by a statistical survey on the socio-economic aspects as well as the fertility of a rural population in Kerala in south-west India. The statistical survey was conducted under the direction of Professor Aleyamma George of the University of Kerala. The sample consisted of 21490 households. Detailed socio-economic, demographic and religious information was collected during the study period which extended over the years 1964 to 1967.

In order to illustrate the use of the measure of "attraction" in socio-anthropological problems we have made use of the data on the number of live births to women in these 21490 households. It is clear from the techniques employed in this article that this measure of "attraction" can also be used for similar situations. Table 1 gives the data for 1964-65 classified according to the various religious groups and villages. The figures appearing in the various columns are the frequencies or the number of live births corresponding to the religious groups and villages. For example there were 288 live births among Nairs in Ulloor village during 1964-65. The figures in brackets are the relative frequencies or the ratios of the frequencies divided by the corresponding total. For example, the total live births among Nairs in 1964-65 is 1239 and hence the relative frequency for Ulloor village is $(288/1239) = 0.2324$. The relative frequencies for Nairs in 1964-65 are $(288/1239, 213/1239, 184/1239, 152/1239, 261/1239, 141/1239)$ and if these are denoted by $(p_1, p_2, p_3, p_4, p_5, p_6)$ then $0 \leq p_i \leq 1, i = 1, \dots, 6$ and $\sum_{i=1}^6 p_i = 1$. Thus in Table 1 we are dealing with

TABLE 1: NUMBER OF LIVE BIRTHS IN THE VARIOUS VILLAGES CLASSIFIED
ACCORDING TO RELIGIOUS GROUPS 1964-65

Religion and caste of the father	Village					Total
	Ulloor	Chetti- vilakom	Vattiurkav	Marukil	Vilappil	Maranalloor
Nairs						
Number	288	213	184	152	261	141
Proportion	(.2324)	(.1719)	(.1485)	(.1227)	(.2107)	(.1138)
Ezhavas						
Number	231	89	69	47	82	65
Proportion	(.3962)	(.1527)	(.1184)	(.0806)	(.1407)	(.1115)
Other Hindus ¹						
Number	233	176	175	103	265	122
Proportion	(.2169)	(.1639)	(.1629)	(.0959)	(.2467)	(.1136)
Catholics						
Number	21	12	3	1	6	5
Proportion	(.4375)	(.2500)	(.0625)	(.0208)	(.1250)	(.1042)
Protestants						
Number	18	11	3	3	4	11
Proportion	(.3600)	(.2200)	(.0600)	(.0600)	(.0800)	(.2200)
Other Christians						
Number	55	86	76	92	169	96
Proportion	(.0958)	(.1498)	(.1324)	(.1603)	(.2944)	(.1672)
Muslims						
Number	20	16	31	1	32	38
Proportion	(.1449)	(.1159)	(.2246)	(.0072)	(.2319)	(.2754)

¹ Backward Communities.

TABLE 2: NUMBER OF LIVE BIRTHS IN THE VARIOUS VILLAGES CLASSIFIED ACCORDING TO RELIGIOUS GROUPS 1965-66

Religion and caste of the father	Village					Total
	Ulloor	Chetti- vilakom	Vattiurkav	Marukil	Vilappil	
Nairs						
Number	260	196	176	142	275	1210
Proportion	(.2149)	(.1620)	(.1455)	(.1174)	(.2273)	(.1331)
Ezhavas						
Number	204	79	60	36	93	556
Proportion	(.3669)	(.1421)	(.1079)	(.0647)	(.1673)	(.1511)
Other Hindus ¹						
Number	227	152	192	100	277	1105
Proportion	(.2054)	(.1376)	(.1738)	(.0905)	(.2507)	(.1421)
Catholics						
Number	23	14	16	—	27	87
Proportion	(.2644)	(.1609)	(.1839)		(.3103)	(.0805)
Protestants						
Number	25	10	8	2	36	91
Proportion	(.2747)	(.1099)	(.0879)	(.0220)	(.3956)	(.1099)
Other Christians						
Number	46	79	48	99	140	546
Proportion	(.0842)	(.1447)	(.0879)	(.1813)	(.2564)	(.2454)
Muslims						
Number	12	11	29	3	33	117
Proportion	(.1026)	(.0940)	(.2479)	(.0256)	(.2821)	(.2479)

¹ Backward Communities.

seven religious groups and we have seven different discrete distributions corresponding to the relative frequencies given in brackets. Closeness of the various communities, as far as live births are concerned, is studied by studying the "attraction" between corresponding discrete distributions. Tables 2 and 3 give the data for the years 1965-66 and 1966-67 respectively.

Table 4 gives the measure of "attraction" among the various communities for the year 1964-65. For example, from Table 1 we have the distribution corresponding to Nairs ($p_1, \dots, p_6$) = (0.2324, . . . , 0.1138) and the distribution corresponding to Ezhavas ($q_1, \dots, q_6$) = (0.3962, . . . , 0.1115). Hence the measure of "attraction" between Nairs and Ezhavas for the year 1964-65 is $\sum_{i=1}^6 2[p_i q_i / (p_i + q_i)] = 0.9651$. The matrix of "attractions" in Table 4 is evidently symmetric and hence only the lower diagonal elements are shown there. The upper diagonal elements can be easily derived since, for example, the attraction between Nairs and Catholics is the same as the attraction between Catholics and Nairs.

By examining the figures in the first column it may be seen that there is maximum attraction between Nairs and Backward Hindus whereas the attraction is least between Nairs and Muslims. In other words, Nairs as a group, as far as the numbers of live births are concerned, behave more like Backward Hindus and least like Muslims. This information is useful if someone is conducting a sample survey to study fertility. These two communities can be combined while taking the samples, or by combining these two communities not much information is lost.

By using Table 4 one can arrange the different communities according to the order of their attraction to other communities. For example, from Column 1 of Table 4 we find that the sequence for Nairs is Nairs, Backward Hindus, Ezhavas, Other Christians, Protestants, Catholics, Muslims.

V Discussion

The main aim of this article is to introduce the measure of "attraction" and illustrate its use to practical problems. By calculating the various "attractions" from Tables 2 and 3 we get tables corresponding to Table 4. Whether the "attraction" between any two communities has changed over the years, these tables should say. If there is a change, one can look into the possible reasons for the change in the group behaviours in the respective communities. These changes may be attributable to changes in the attitudes or religious observances or other anthropological characteristics. In a religion-dominated society such a study will be interesting. Kerala communities were such that taboos, customs, costumes and even food habits were different for different communities. This situation is rapidly changing. The measure of "attraction" introduced in this article will be helpful in detecting such changes by studying "attraction" over the years.

This measure may also be used in studying other socio-economic characteristics as well. It is seen from the expressions that once we form frequency tables, the data can be analyzed to a great extent by using this measure of "attraction."

A statistical problem that can be looked into is that of making some sort of probability statements on this measure of "attraction." For what values of this measure are the two groups "significantly" close or "significantly" different? An examination of the sampling distribution of this measure might provide an answer.

The exact sampling distributions of many such measures are still open problems. Some applications and a lot of literature are available on Matusita's measure of affinity and some measures in Information Theory. Similar results on the measure of "attraction" as well are noted. References to the literature in this direction are available from Mathai and Rathie (1975). Also one could look into the behaviour of A_n as a function of n —the number of

TABLE 3: NUMBER OF LIVE BIRTHS IN THE VARIOUS VILLAGES CLASSIFIED ACCORDING TO RELIGIOUS GROUPS 1966-67

Religion and caste of the father	Village					Total
	Ullloor	Chetti-vilakom	Vatturkav	Marukil	Vilappil	Maranalloor
Nairs						
Number	231	191	171	104	197	143
Proportion	(.2228)	(.1842)	(.1649)	(.1003)	(.1900)	(.1379)
Ezhavas						
Number	235	59	49	40	64	70
Proportion	(.4545)	(.1141)	(.0948)	(.0774)	(.1238)	(.1354)
Other Hindus ¹						
Number	206	135	163	82	207	167
Proportion	(.2146)	(.1406)	(.1698)	(.0854)	(.2156)	(.1740)
Catholics						
Number	16	23	25	1	38	10
Proportion	(.1416)	(.2035)	(.2212)	(.0088)	(.3363)	(.0885)
Protestants						
Number	11	26	18	1	47	8
Proportion	(.0991)	(.2342)	(.1622)	(.0090)	(.4234)	(.0721)
Other Christians						
Number	44	20	23	88	58	124
Proportion	(.1232)	(.0560)	(.0644)	(.2465)	(.1625)	(.3473)
Muslims						
Number	21	11	35	1	19	32
Proportion	(.1765)	(.0924)	(.2941)	(.0084)	(.1597)	(.2689)

¹ Backward Communities.

TABLE 4: ATTRACTION BETWEEN POPULATIONS, 1964-1965

	Nairs	Ezhavas	Backward Hindus	Catholics	Protestants	Other Christians	Muslims
Nairs	1.0000						
Ezhavas	.9651	1.0000					
Backward Hindus	.9962	.9549	1.0000				
Catholics	.8965	.9604	.8871	1.0000			
Protestants	.9075	.9560	.8907	.9601	1.0000		
Other Christians	.9558	.8620	.9592	.7701	.8153	1.0000	
Muslims	.8912	.8461	.9117	.7914	.8296	.8939	1.0000

partitions or the conditions under which A_n is equal to A_{n-k} for $k = 1, 2, \dots$. It should be pointed out that A_n is a measure to study the "closeness" of groups once the categorization is made. It does not deal with the categorization of the data itself. But the analysis may throw some light on the desirability of combining certain groups. This aspect is pointed out in the last but one paragraph of section 4.

The term "attraction" is used as a synonym to the term "affinity." This is not a distance measure but it measures closeness of the groups under consideration in the sense of the measure of "affinity."

The aim of this paper is to point out the measure and to illustrate one of its possible applications. A complete study of its mathematical properties and practical applications is not attempted here. Very little work has been done in this direction.

References

- Kaufman, H. and A. M. Mathai. 1973. A Mathematical Foundation for the Measure of Affinity. *Journal of Multivariate Analysis* 3:236-242.
- Mathai, A. M. 1974. Characterizations of Some Statistical Concepts. *Proceedings of the Conference on Measures of Information and Their Applications*. Bombay: Indian Institute of Technology.
- Mathai, A. M. and P. N. Rathie. 1971. *Basic Concepts in Information Theory and Statistics: Axiomatic Foundations and Applications*. New York: Wiley Holsted, New Delhi: Wiley Eastern.

Received September, 1974; Revised August, 1975.