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ON THE TIME REQUIRED FOR ATTAINING THE DESIRED SIZE AND SEX COMPOSITION OF THE FAMILY

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Résumé — On propose ici un modèle mathématique simple pour l'étude de la variation dans le temps requis pour arriver à la dimension et la composition par sexe de la famille en prenant la durée de mariage du couple comme finie. Aux fins d'illustration on a considéré quatre cas hypothétiques appelés règles d'arrêt et pour chaque cas, le temps escompté aussi bien que la proportion des couples satisfaits ont été calculés pour produire différentes combinaisons de durées maritales, de niveaux de fécondabilité et des périodes de repos (durée de gestation et durée d'aménorrhée). On a aussi obtenue une estimation du biais de troncation pour chaque cas.

Abstract — A simple mathematical model has been proposed to study variation in the time required for attaining the desired size and sex composition of the family by taking marital duration of the couple as finite. For illustration four hypothetical cases, termed "stopping rules," have been considered, and for each case the expected time along with the proportion of satisfied couples have been computed for different combinations of marital durations, levels of fecundability and the rest periods (gestation period plus amenorrhea period). Also, estimate of the truncation bias has been obtained for each case.

Key Words — **sex preference, satisfied couples, stopping rules, family building process.**

I. Introduction

The family decision process is a complex phenomenon and is not as simple as other physical and biological processes are. Several factors, *viz.*, number of living children; the sex composition; socio-economic status; cultural norm; utility of child bearing; subjective valuation of contraception, etc., act simultaneously in decision making. A couple may decide at a point of time to interfere in some way with the birth process. The consequence of this decision is a chance phenomenon leading to a finite size and sex composition of the family. An aid to decision making at the micro level would be the knowledge of the expected time during which the desired family size along with the desired sex composition could be achieved — this information would be useful for couples in planning their family in such a way that their social and economic liabilities are over before they retire from an active life. The utility of contraception may also be better realized by individual couples if they are aware of the chance mechanism of the family building process and the expected time to achieve the desired size and sex composition of the family. However, at macro level, the interest may lie in having an estimate of the number of those couples whose desire with respect to size and sex composition has been fulfilled in a given time (hereafter we shall refer to such couples as satisfied couples). This may be useful in projecting the future number of the satisfied couples who may require family planning services and also in formulating the national population policy.

Bearing in mind the utility of information we need and the complexity that it poses, we have developed a mathematical model which may help answering the related queries at micro as well as at macro level without involving much mathematical intricacy. For illustration, four hypothetical cases, termed "stopping rules," have been considered and for each case the expected time for attaining the desired family size and the proportion of satisfied couples have been computed for various combinations of marital durations, levels of fecundability and the rest periods (gestation period plus amenorrhea period). Also, estimate of the truncation bias (introduced due to short marital duration) has been obtained for each case.

II. The Problem and Its Analogy

Several KAP studies have revealed that in less developed countries, where the level of literacy is low, the "ideal family size" reported by a couple is the function of the size they actually have had at the time of the survey. True option regarding the "desired family size" is observed to be time dependent and is subject to the change in utility of child bearing to the couple either because of the sex composition achieved or because of the change in socio-economic status. In a survey, it may be difficult to ascertain ideal family size and desired family size from the response of couples.

This may, however, be ascertained if it could be known what is the desired sex composition of children aimed at by the couples. Keeping this in view, Sheps (1963) used the well-known results of the coin-tossing experiment to show the effect of sex preference on the completed size by taking the reproductive life of a woman as infinite. However, it is not always possible for a couple to achieve the desired family size because (i) the duration of a woman's fertile period is limited, (ii) the conceptions are, to some extent, random events; (iii) a part of the effective marital duration of a woman is wasted owing to gestation and amenorrhea periods following every conception and (iv) a couple may stop voluntarily — this may be because of the number and sex composition of the born children and a feeling of the impossibility of achieving the desired composition. Thus, the conditions of a coin-tossing experiment as considered by Sheps (1963) and Mitra (1970) are not completely fulfilled by the process of reproduction. With this consideration, Pathak (1973) proposed a model to study the variation in the family size attained in a given time under different parental preferences regarding the sex of children. Following a similar approach, we have formulated a model for the waiting time of realization of such an event. Of course, its generalized version could have been obtained on the lines of Das Gupta (1973) and Mode (1975). The expressions may, however, be treated as an approximation of the reality.

The demographic problem envisaged in this paper may be compared with the one of the life testing in case of some engineering products, where the experimenter is interested to observe n articles for some time T and to stop the experiement as soon as either $r \leq n$ failures occur or time T is reached. One of the variables of interest is the waiting time for the r^{th} failure. The problem necessarily reduces to finding out the waiting time distribution, which is both truncated in time and censored for the number of failures. But in the present case, the problem of obtaining the waiting time distribution is not so simple: after a conception there is no possibility of another conception for some time owing to the woman's rest period; the moment she is relieved from the confinement and the associated amenorrhea period, she is again exposed to the possibility of conception and her waiting time starts. These states are recurrent. In case of engineering goods, however, it is not the same article failing again and again, but out of n articles, the experimenter is interested in r failures before time T. Here our interest lies in finding the waiting time for realizing any one of the conditions satisfying the desire of the couple in respect of its family size before time T. The distribution obtained here may also be used in analogous cases of life testing.

III. The Model

The underlying assumptions of the model are as follows:

1. A woman is not pregnant and is fecund at the time of consummation of her marriage and continues to be in marital union until time T.

2. The probability that $(i+1)^{th}$ conception occurs during the time interval $(t, t+dt)$ is $\lambda\, dt + o\,(dt)$ if the i^{th} conception occurs prior to $t-h$, where $t \geq ih$; $\lambda > o$, and zero otherwise; h is the rest period associated with a live birth conception.

3. Let $(1 - \alpha_i)$ be the probability that a woman becomes secondary sterile after attaining the i^{th} *parity*.

4. The probability that a born child will be male is g and female $(1\text{-}g)$. The probability of twin or multiple births is zero.

5. A couple decides to stop whenever any one of the following conditions are fulfilled: (i) either a sons and b daughters are born, or (ii) m $(m > a+b)$ children are born.

Some of these assumptions are no doubt strong but they may be considered as the first approximation to the real process.

Let x denote the time required by a couple to attain the desired size and sex composition of the family. Suppose this is achieved at parity i, then the joint distributions of x and i is given by:

$$f(x,i) = \prod_{j=1}^{i-1} \alpha_j \frac{\lambda^i}{(i-1)!} \left[x-G-(i-1)h\right]^{(i-1)} exp\left[-\lambda\left\{x-G-(i-1)h\right\}\right] \cdot \left[\binom{i-1}{a-1} g^a (1-g)^{i-a} + \binom{i-1}{b-1} g^{i-b} (1-g)^b\right] \quad \ldots(1)$$

where, $x \geq (i\text{-}1)h + G$

and $i = (a+b), (a+b+1), \ldots, (m\text{-}1)$

G is gestation period.

Now, since the couple stops whenever m children are born according to assumption (ii), the joint distribution of x and m is given by:

$$f(x,m) = \prod_{j=1}^{m-1} \alpha_j \frac{\lambda^m}{(m-1)!} \left[x-G-(m-1)h\right]^{(m-1)} exp\left[-\lambda\left\{x-G-(m-1)h\right\}\right] \cdot \left[\sum_{k=0}^{a-1} \binom{m-1}{k} g^k (1-g)^{m-k-1} + \sum_{k=0}^{b-1} \binom{m-1}{k} g^{m-k-1} (1-g)^k\right] \quad \ldots(2)$$

where, $x \geqslant (m-1)h+G$

The proof of the results is given in the Appendix.

The mean waiting time, $\bar{x}$ is:

$$\bar{x} = \frac{\sum_{i=(a+b)}^{m-1} \int_{(i-1)h+G}^{T} x f(x,i)\,dx + \int_{(m-1)h+G}^{T} x\, f(x,m)\,dx}{\sum_{i=(a+b)}^{m-1} \int_{(i-1)h+G}^{T} f(x,i)\,dx + \int_{(m-1)h+G}^{T} f(x,m)\,dx} \quad \ldots.(3)$$

It may, however, be noted that here X is not a proper random variable because all the couples cannot attain their desired family size within the period (O,T) and hence the denominator in the expression (3) gives the proportion of those couples who can achieve the desired family size under assumption 5. For numerical values of T, h, G and m, both numerator and denominator can easily be ascertained after actually integrating definite integrals.

The variance of the waiting time X can be found by finding the second moment (M_2) about the origin, as below:

$$M_2 = \frac{\sum_{i=(a+b)}^{m-1} \int_{(i-1)h+G}^{T} x^2 f(x,i)\,dx + \int_{(m-1)h+G}^{T} x^2 f(x,m)\,dx}{\sum_{i=(a+b)}^{m-1} \int_{(i-1)h+G}^{T} f(x,i)\,dx + \int_{(m-1)h+G}^{T} f(x,m)\,dx} \quad \ldots(4)$$

When $T = \infty$, expression (3) reduces to

$$\frac{\sum_{i=(a+b)}^{m-1} \left\{\frac{i}{\lambda} + G + (i-1)h\right\} P_i + \left\{\frac{m}{\lambda} + G + (m-1)h\right\} P_m}{\sum_{i=(a+b)}^{m-1} P_i + P_m} \quad \ldots(5)$$

The values of P_i and P_m can be computed from the expressions (A.1) and (A.2) respectively, given in the Appendix.

Also, for $T = \infty$, the expression (4) reduces to:

$$\frac{\sum_{i=(a+b)}^{m-1} \left[\frac{i(i+1)}{\lambda^2} + \frac{2i\left\{G+(i-1)h\right\}}{\lambda} + \left\{G+(i-1)h\right\}^2\right] P_i}{\sum_{i=(a+b)}^{m-1} P_i + P_m} +$$

$$+ \frac{\left[\frac{m(m+1)}{\lambda^2} + \frac{2m\{G+(m-1)h\}}{\lambda} + \{G+(m-1)h\}^2\right] Pm}{\sum_{i=(a+b)}^{m-1} P_i + P_m} \qquad \ldots(6)$$

Expressions (5) and (6) have been derived by first finding the $E(x/i)$ and $E(x^2/i)$ and then by taking the expectations of these conditional measures.

IV. Illustrations

Four hypothetical cases, giving the rules when a couple would stop, have been considered, and in each case the expected time for attaining the desired family size and the proportion of satisfied couples have been computed for various combinations of marital duration (T), level of fecundability (λ), and the rest period (h). These four cases, however, are arbitrary and are only illustrative. Several other stopping rules may be framed, and the expected waiting time and the proportion of satisfied couples can be computed. But we restrict the present analysis to the following four cases only:

Rule 1: A couple stops when at least one son or three children are born ($a=1$, and $m=3$).

Rule2: A couple stops when at least one son and one daughter or three children are born ($a=1$, $b=1$ and $m=3$).

Rule 3: A couple stops when at least two sons and one daughter or four children are born ($a=2$, $b=1$, and $m=4$).

Rule 4: A couple stops when four children are born ($m=4$).

Using the expressions (1), (2) and (3) given in the previous section, the expected values of the waiting time for fulfilling the desire of couples in the above cases have been computed corresponding to T = 10 and 15 years, h = 1.25 or 1.50 years and λ = 0.40 or 0.60. The values of λ have been chosen arbitrarily but are consistent with the empirical estimates obtained in Singh (1968), Singh and Pathak (1968) and Saxena (1969) for Indian women. Also, the justifications for the values of h (nine months of gestation plus six months or nine months of postpartum amenorrhea) are given in Saxena (1966). For simplicity we have assumed $g = ½$. Though in reality $g \neq ½$ and also may vary from parity to parity, for the computation of the expected waiting time and the proportion of satisfied couples, $g = ½$ is a reasonable approximation since a slight deviation from this value will not have significant effect on the results. Further, we have assumed $\alpha_i = \alpha$ and the empirical value of α is taken as 0.96 as obtained by Pathak (1975). The expected waiting time and the proportion of satisfied couples under different sex preferences and stopping rules are presented in Table 1.

The standard deviations of the waiting time have been computed under different rules for $T = \infty$, from the expressions (5) and (6). The results are presented in Table 2. It may be mentioned, however, that the standard deviation of the waiting time under a finite marital duration would be less than its corresponding value computed for $T = \infty$.

V. Truncation Bias

Sheps (1963) studied the effect of sex preference on the completed family size by taking the reproductive life of a woman as infinite. However, the duration of a woman's fer-

TABLE 1 EXPECTED WAITING TIME AND PROPORTION OF SATISFIED COUPLES UNDER DIFFERENT SEX PREFERENCES AND STOPPING RULES

Marital Duration T (In Years)	Fecund-ability λ	Rest Period h (In Years)	Expected Waiting Time $\bar{x}$ (in years) and Proportion (Given in Parenthesis) of Satisfied Couples Under			
			Rule 1	Rule 2	Rule 3	Rule 4
10	0.40	1.25	4.34 (0.8032)	6.39 (0.6312)	7.86 (0.2749)	8.51 (0.1598)
		1.50	4.37 (0.7883)	6.56 (0.6014)	8.05 (0.2270)	8.77 (0.1109)
	0.60	1.25	3.95 (0.9039)	5.87 (0.8116)	7.59 (0.4978)	8.27 (0.3713)
		1.50	4.03 (0.8916)	6.11 (0.7871)	7.85 (0.4263)	8.60 (0.20124)
15	0.40	1.25	5.36 (0.9253)	7.87 (0.8540)	10.38 (0.6273)	11.28 (0.5349)
		1.50	5.46 (0.9198)	8.14 (0.8430)	10.73 (0.5879)	11.68 (0.4838)
	0.60	1.25	4.42 (0.9629)	6.60 (0.9260)	9.32 (0.8188)	10.25 (0.7729)
		1.50	4.58 (0.9611)	6.93 (0.9225)	9.81 (0.7949)	10.79 (0.7387)
∞	0.40	1.25	5.96 (0.9704)	8.84 (0.9408)	13.06 (0.8986)	14.50 (0.8847)
		1.50	6.14 (0.9704)	9.21 (0.9408)	13.71 (0.8986)	15.25 (0.8847)
	0.60	1.25	4.52 (0.9704)	6.76 (0.9408)	10.04 (0.8996)	11.17 (0.8847)
		1.50	4.70 (0.9704)	7.13 (0.9408)	10.70 (0.8986)	11.92 (0.8847)

tile period is limited and, therefore, to regard it as finite would be more reasonable. But, again, for a short marital duration only those couples will be satisfied, in regard to the desired size and sex composition of the family, whose female spouses are quick conceivers, and thus a bias is introduced in the estimate of the expected waiting time — the shorter the marital duration, the greater the extent of bias (Sheps *et al.*, 1970). To make the results given in Table 1 more meaningful, it is desirable to study the effect of truncation of marital duration upon the expected waiting time. The estimate of the extent of the bias in each case is obtained by subtracting the value of the expected waiting time computed for the finite marital duration (T) from the value computed for the infinite marital duration for the given set of values of the parameters λ and h (Table 3).

TABLE 2 STANDARD DEVIATIONS OF WAITING TIME OF SATISFIED COUPLES WITH INFINITE MARITAL DURATION UNDER DIFFERENT SEX PREFERENCES AND STOPPING RULES

Fecundability λ	Rest Period h (In Years)	Standard Deviation of Waiting Time (In Years) of Satisfied Couples Under			
		Rule 1	Rule 2	Rule 3	Rule 4
0.40	1.25	4.50	4.36	5.09	5.00
	1.50	4.64	4.42	5.14	5.00
0.60	1.25	3.25	3.01	3.49	3.32
	1.50	3.40	3.08	3.52	3.32

TABLE 3 ESTIMATES OF TRUNCATION BIAS IN THE EXPECTED TIME FOR ATTAINING THE DESIRED FAMILY SIZE UNDER DIFFERENT SEX PREFERENCES AND STOPPING RULES

Marital Duration T (In Years)	Fecundability λ	Rest Period h (In Years)	Truncation Bias (In Years) in the Expected Waiting Time Under			
			Rule 1	Rule 2	Rule 3	Rule 4
10	0.4	1.25	1.62	2.45	5.20	5.99
		1.50	1.77	2.65	5.66	6.48
	0.6	1.25	0.57	0.89	2.45	2.90
		1.50	0.67	1.02	2.85	3.32
15	0.4	1.25	0.60	0.97	2.68	3.22
		1.50	0.68	1.07	2.98	3.57
	0.6	1.25	0.10	0.16	0.72	0.92
		1.50	0.12	0.20	0.89	1.13

VI. *Discussion and Conclusions*

It can be seen that the expected waiting time for attaining the desired family size is more under Rule 4 when compared with the same in the rest of the three hypothetical cases illustrated here (Table 1). In this case, no sex preference is indicated and the desire of a couple is fulfilled when four children are born. Under Rule 3, where greater preference for boys is shown, a couple may stop reproduction even after three children provided the desired sex composition in the family is achieved, and hence the expected waiting time is less than the same for getting four children. For Rule 2, however, where a couple gives equal preference for a child of each sex, the expected waiting time for getting the satisfaction is still less than the expected waiting time obtained under Rules 3 and 4. It is at least for Rule 1, where the desire for a girl is not shown and a couple may stop even when one son is born. The proportion of satisifed couples is greater for Rule 1 and this proportion as we go from Rule 1 to Rule 4 for a given set of values of the parameters λ and h and for a fixed marital duration T.

Table 1 also shows that the couples whose female spouses are more fecund and for whom the duration of postpartum amenorrhea is less, achieve satisfaction with respect to the size and sex composition of the family sooner than those with lower fecundability and a higher duration of postpartum amenorrhea. From Table 2, it is clear that the extent of bias due to truncation is lesser in case of those couples whose female spouses are more fecund and for whom the average duration of postpartum amenorrhea is shorter. Also, the higher the desired size of the family, the greater is the extent of the bias. For example, the bias is least under Rule 1 when $\lambda = 0.60$ and $h = 1.25$ years, for a given marital duration T. This is because the requirement in this case is only of having a son and hence majority of couples have their desires fulfilled much earlier, while only a smaller proportion remains unsatisfied. In cases of more stringent conditions regarding the size and sex composition of the family, a comparatively smaller proportion of the couples have their desires fulfilled. Obviously, those couples whose female spouses are quick conceivers are included and hence the extent of bias due to truncation is larger in more stringent cases.

The probable range of the expected waiting time for fulfilling the desire of a couple with respect to the size and sex composition of the family can easily be read from Table 1: the lower limit of the range in each case is constituted by that value of the expected waiting time which corresponds to the pair of parameters $\lambda = 0.60$ and $h = 1.25$ years. The upper limit is that value of the expected waiting time which is obtained by assuming $\lambda = 0.40$ and $h = 1.50$ years for an infinite marital duration. Also, the proportion of satisfied couples in each case and in a particular period of time can be ascertained. This information may be useful in projecting the future number of the satisfied couples who may require family planning services. Thus, more precise estimates of the future needs of family planning services can be obtained if all marriages are registered and at the time of registration the desires of the couple regarding the size and sex composition of the family which they would like to have is recorded. This *a priori* information along with the expected waiting time of having the desire fulfilled would certainly help in identifying those couples who may need family planning services at a particular time or during a period of time. This selective approach would not only minimize the operational cost of family planning administration but also would maximize the returns in terms of sterilization, number of IUD insertions, etc., in a comparatively shorter time and in a more efficient manner.

Thus, the proposed model may help in understanding the underlying chance mechanism of the family building process and may provide an aid to decision making at the micro level. At macro level, it may give an estimate of the population of satisfied couples who may need family planning services in future. It may be mentioned, however, that the model derived here is based on some strong assumptions which may be relaxed to make it more general and of more practical utility.

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Appendix

Following Sheps (1963), the probability that the desire of the couple is fulfilled at the birth of the i^{th} child is given by:

$$P_i = \left[\binom{i-1}{a-1} g^a (1-g)^{i-a} + \binom{i-1}{b-1} g^{i-b} (1-g)^b\right] \qquad \ldots(A.1)$$

The probability that the desire of a couple with respect to the size and sex composition of family is not fulfilled till the birth of $(m\text{-}1)^{th}$ child and consequently the woman stops reproduction when m^{th} child is born, is given by:

$$P_m = \left[\sum_{k=0}^{a-1} \binom{m-1}{k} g^k (1-g)^{m-k-1} + \sum_{k=0}^{b-1} \binom{m-1}{k} g^{m-k-1} (1-g)^k \right] \qquad \ldots (A.2)$$

Again, using Assumption 3 and results of Singh (1968), we have the conditional distribution of x given i ($i = 1, 2, \ldots, m$-1)as:

$$\frac{\lambda^i}{(i-1)!} \left[x - G - (i-1)h \right]^{(i-1)} \exp\left[-\lambda \left\{ x - G(i-1)h \right\} \right] \prod_{j=1}^{i-1} \alpha_j \qquad \ldots (A.3)$$

Hence, the joint distribution function of x and i is the product of (A.1) and (A.3) or (A.2) and (A.3) depending upon the range of i. In case x is truncated at T, the joint densities given by equations (1) and (2) will be divided by the denominator of equation (3).