

Migration and Population Replacement

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Abstract

Three stable population models are described. The first two are closed to migration, with fertility fixed at the current level and at replacement. The third has fixed amounts of migration, with fertility at the current level. An alternative using fixed migration rates is considered. The models are exemplified with current Canadian data, for the first century and ultimately, paying special attention to age distribution. The role of migrant age is examined. Replacement by migration is an acceptable alternative to replacement by fertility, and much preferable to a closed model with current fertility. Limitations on the usefulness of the exercise for policy are assessed.

Résumé

Trois modèles de population stables sont décrits. Les deux premiers sont fermés à la migration, le taux de fécondité se maintenant au niveau actuel et de reproduction. Le troisième a des volumes fixes de migration et se maintient au niveau actuel de la fécondité. Un autre modèle fondé sur des taux de migration fixes est envisagé. Les modèles sont illustrés à l'aide de données Canadiennes actuelles, pour le premier siècle et accorde une attention particulière à la répartition des âges. Le rôle de l'âge des migrants est examiné. La reproduction par la migration offre une alternative acceptable à la reproduction par la fécondité, et constitue une solution largement préférable à un modèle fermé au taux de fécondité actuel. Les limites de l'utilité de cet exercice sur le plan politique sont examinées.

Key Words: *Stable Models; Migration; Age Distribution; Canada*

Introduction

During the past several decades the net reproduction rate for Canada has been well below the replacement level, yet there is no sign of imminent decline in population size. The rate of growth exceeds one percent and has varied little over recent time. In brief, the prediction implied by the conventional stable population model is an inappropriate characterization of current population change. The reason is that the model is closed to migration, and Canada is not.

Reluctance to incorporate migration in stable population models is understandable because that process of population change has historically been much more variable over time than fertility or mortality. Nevertheless with the prospect of persistent subreplacement fertility in a context favorable to immigration, it seems appropriate to reformulate the conventional model. The supply of immigrants is unlikely to slacken in the foreseeable future, given the many large populations with rapid growth rates and levels of development well below that of Canada. The demand for immigrants will be stimulated by the desirability of avoiding a future in which the population is older and smaller.

In what follows, three population models are described, and exemplified with current data for Canada. The first is the conventional stable model, closed to migration, with fertility and mortality held fixed henceforth at their current levels. The other two models are based on assumptions that guarantee ultimate stationarity, with the same ultimate population size. The former achieves that objective by an adjustment of fertility upward to the replacement level but with no migration; the latter leaves fertility fixed at its current level but assumes a constant inflow of net migrants of just the amount calculated to culminate in the same outcome.

The focus of the analysis is comparative age distributions. The reason for this choice is that it is difficult to arrive at an objective judgment of an optimal future size for the population – other than that it should eventually become stationary – whereas it is relatively easy to make judgments about the comparative desirability of different age distributions.

Two issues require particular attention. In the first place, a choice must be made between two credible assumptions about age-specific net migration: either fixed in absolute numbers or fixed relative to the size of the receiving population. Although the choice here is the former, the decision is not clear-cut. In the second place, the outputs for comparative analysis – various parameters of the age distribution – are quite sensitive to the assumed age distribution of net migrants. The question calls for separate scrutiny. The account is concluded with an assessment of the various respects in which the analysis, while broadly suggestive, remains insufficient for policy guidance.

Section 1. Closed and Open Population Models.

This section contains an account of the equilibrium properties of various population models, all of them stable because they are based on assumptions of permanent fixity of the processes of population change: fertility, mortality and net migration. Life tables for males and females have been constructed for Canada, based on the age-specific mortality rates observed in 1994. (Statistics Canada, 1996). In this section, however, both the account and its illustrative data apply solely to females. (The Lotka model on which the work is founded was, after all, a one-gender model.) Subsequent derivation of parallel results for males would be a simple subsidiary exercise, adding no formal contribution. The fertility rates are those of 1994; the amounts and rates of net migration by age are for 1994-95. If one views the models as projections, the starting-point is the population size, by age and gender, as of July 1, 1995 (Statistics Canada, 1996).

The first model is a population closed to migration, with current age-specific fertility and mortality. The net reproduction rate is 0.79612, and the intrinsic rate of natural increase, estimated by the Wicksell procedure, is $r = -0.008075$. (Keyfitz, 1968, pp. 147-149.) Thus, this is a subreplacement population with a rate of decline approaching one-half every 86 years. Although the population ultimately vanishes, one can examine its equilibrium age distribution by calculating the function $\exp(-rx) \cdot S_x$. (The symbol, S_x , for survivors to age x , is used in preference to the more common symbol, to avoid confusion between the letter "l" and the number "1".)

For heuristic purposes, it is worthwhile considering also the outcome for a population, likewise closed to migration, in which the net reproduction function is adjusted upward proportionately, so that the net reproduction rate is 1.0, and thus $r=0$. Elsewhere (Ryder, 1975) it has been shown how to determine directly the ultimate size of birth cohort for such a model, but in continuous form; here the formula is converted to discrete calculations. Designate the elements of net reproduction by $R_{5y} = F_{5y} \cdot S_{5y}$, where F_{5y} is female fertility and S_{5y} is female survival, for ages $y=3, 10$, i.e., centered on ages 15, 20, ..., 50. To raise the observed net reproduction rate to 1.0, calculate

$$R_{5y} = R_{5y} / \sum_{y=3}^{10} R_{5y}$$

This is of course only one of an infinity of ways in which replacement reproductivity could be obtained. Moreover it is not unlikely that, if fertility were higher than it is currently, its shape by age would differ. The choice here is arbitrary but simple, and it ordinarily makes only a small empirical difference from alternatives.

In finite form, the expression for ultimate birth cohort size, say U,

$$U = \left(\sum_{x=1}^{10} \left[\left(N_{5x} / S_{5x} \right) * \sum_{y=x}^{10} R'_{5y} \right] \right) / \left(\sum_{x=1}^{10} \sum_{y=x}^{10} R'_{5y} \right)$$

where N_{5x} , the number of females in the population at the outset, in exact ages 5, 10, ..., has been estimated as $[{}_5N_{5(x-1)} + {}_5N_{5x}]/10$. The result for Canadian females, 1994, is $U=204,477$. With the ultimate cohort size as radix, and the survival function, S_x , one can determine the age distribution at equilibrium, and the total population size. The latter is simply the product of U and the female expectation of life at birth $e_o = 81.083$. The ultimate female population size is 16.823 million, compared with the current 14.942 million. The increase in size is of course a manifestation of population momentum.

Models of the future based on fixed processes of population change over time cannot pretend to be realistic. They do however serve the descriptive purpose of showing the structural consequences of current behavior. They also have a prescriptive function, by illustrating the relationships between various courses of action and the goals of population policy.

To abandon the closed population assumption, and introduce net migration into the stable population model, the procedure analogous to the closed model is to calculate age-specific net migration rates, say ${}_5n_{5x}$, directly comparable in form to mortality rates, ${}_5m_{5x}$. (See Hyrenius, 1959; Lopez, 1961; Pollard, 1973). Calculate a variable for the rate of change in cohort size, say ${}_5g_{5x} = {}_5n_{5x} - {}_5m_{5x}$. Then ${}_5g_{5x}$ can be used to create a cohort growth function, say G_{5x} , in exactly the same way as ${}_5m_{5x}$ is used to create a survival function, S_{5x} . Thenceforth the development of the model follows the conventional form, except that G_{5x} is substituted for S_{5x} . (For summaries of the literature, see Espenshade, 1986; Sivamurthy, 1982).

One important empirical consequence of this reformulation is that there is a different net reproduction rate, 1.00775, and a different intrinsic rate of increase, $r=+0.0027134$. Accordingly, if we take into account not only fertility and mortality but also net migration, the population of Canada is not intrinsically declining but rather growing – albeit at a very slow pace. Interjection of net migration into the calculations has increased net reproductivity, in this more comprehensive sense, by more than 26 percent. If the net reproduction rate is the relative numbers of female births, one generation apart, it increases because of the contribution of net migrant females to the birth output.

In the remainder of this section, we propose to reduce the net migration rates slightly from the observed values, in order to bring the intrinsic rate of natural

increase and migratory increase to zero. Although again there are infinitely many ways to do this, the task is most simply accomplished by reducing each age-specific net migration rate by the value r . The principal purpose of this modification is to achieve a determinate ultimate population size. From a population policy standpoint, intrinsic growth (or decline) may be considered more desirable in the short or intermediate term, but stationarity is necessary in the very long term.

With the migration equivalent of the net reproduction rate set at unity, the value for ultimate birth cohort size can be found with the same formula as provided above, by substituting the cohort growth function, G_{5x} , for the survival function, S_{5x} . The outcome is $U=182,459$. This is a substantially smaller value than that obtained for the closed replacement fertility model because the growth values (G_{5x}) used as divisors of the current female population size are appreciably larger than the survival values (S_{5x}). Another source of discrepancy, much smaller, is that the age distribution of the modified net reproduction function is somewhat older than that of the original.

The second important empirical consequence of using the cohort growth function in place of the survival function is that the cohort population size over a lifetime (the equivalent of the conventional expectation of life at birth of 81.083) is much larger, here 109,432. The ultimate (female) population size, the product of this value and the new ultimate cohort size of 182,459, becomes 19.967 million, which is 19 percent larger than in the replacement fertility model. This outcome is understandable because, in effect, the objective of replacing the population has been accomplished by the importation of a substantial additional population of immigrants.

Section 2. Two Migration Models

An alternative to this "rate" model for incorporating immigration in stable population theory had been described by Espenshade et al. (1982). They showed that, with fertility and mortality fixed, and net reproductivity below replacement, a regime of fixed numbers of immigrants by age and gender each year will eventuate in a stationary population. (Pollard, 1973, showed the stability properties of this model. Mitra, 1983, generalized the argument for any level of net reproduction. See also Coale, 1972). In the following presentation, we take the liberty of modifying their account with respect to symbols and procedures; the substance of their argument is unaffected.

To determine the characteristics of the ultimately stationary population based on the "amount" model, the first step is to project the numbers of net migrants by age (modified appropriately by survival) to their final cumulated shape, some hundred years hence. For Canada, 1994-95, the number of female migrants per annum, $M=97,079$, and the size of the migrant population eventually generated is 5.047 million. For further development, it helps to note that the ratio of the latter to the former, the average expectation of life after migration, is $EM = 51.99$.

With the completed female migrant population by age, and the conventional age-specific fertility rates, conjoined in a product-sum, we can obtain the equilibrium number of births to migrants, here 38,145. Again it is helpful for further development to express this value relative to the number of migrants (97,079), giving the average reproductive value of a migrant, say $RM = 0.3929$.

The basic formula for determining the ultimate birth cohort size (derived from Espenshade et al., 1982) is $U = M \cdot RM / (1 - R_0) = 187,095$. The ultimate population consists of the product of $U \cdot e_0$ and the product of $M \cdot EM$, for a total of 20.217 million. The proportion of the population migrant is 25 percent (a value independent of the number of migrants per annum).

Although this procedure differs from that used with the "rate" model, it is instructive to reconsider the latter in light of the former. The "rate" model depends on ultimate birth cohort size— a weighted average of initial population size by age, where the weights are provided by the cohort growth function. To infer the critical parameters RM , the reproductive value of a woman, and EM , the expectation of life after migration, one can proceed as follows. With the ultimate population by age, and the age-specific migration rates, the product-sum gives the number of migrants $M=107,302$. Recasting the formula for ultimate birth cohort size given above for the amount model, one has

$$RM = (U/M) \cdot (1 - R_0) = 0.3467.$$

It is acceptable to use this formula because, at equilibrium, the number of migrants per annum, as well as the rates, are fixed. Furthermore, the total migrant population size can be found by subtracting from total population size the product of ultimate cohort size and the expectation of life at birth. When this is divided by the number of migrants, one obtains for the expectation of life after migration a value of 48.22.

The parameter RM is larger for the amount model (0.3929) than for the rate model (0.3467). Likewise the parameter EM is larger for the amount model (51.99) than for the rate model (48.22). Both differences reflect the comparative age distribution of migrants, the mean of which is 30.4 for the amount model and 34.5 for the rate model. Younger migrants have higher reproductive value, and live longer subsequent to migration. In the amount model the age distribution of migrants is fixed by assumption, but in the rate model, although it starts at that value, it eventually becomes much older because the population ages during its evolution toward equilibrium.

The critical formal difference between the two models is that the amount model begins with a specified number of migrants, M , and then the value of ultimate birth cohort size, U , is inferred from an equation in which the strategic variable is the reproductive value of a migrant, RM . The rate model begins with the value U , and the value M is inferred (from the same formula in reverse). Thus the amount model specifies the size of birth cohort that can be supported at stationarity by a given number of net migrants, whereas the rate model specifies

the number of net migrants required to support a given size of birth cohort at stationarity.

The rate model eventuates in a stable equilibrium, of which the stationary (the one discussed here) is a special case. The amount model, regardless of the number of migrants annually, always leads to a stationary equilibrium (provided net reproductivity is below replacement). The rate model, provided it is calibrated to replacement, culminates in one particular population size, determined in large part by the age distribution of the initial population. The amount model, on the contrary, can be scaled to achieve any specified population size – perhaps in response to policy considerations – simply by modifying appropriately the annual number of migrants. The initial population is in fact irrelevant; the model could be used to describe the populating of a previously empty land.

There is, however, a particular limitation on the amount model. The question addressed (in Espenshade et al., 1982) is not the consequence of a specified amount of net migration but rather of a specified amount of immigration. The authors reveal ambivalence about what their subject is, using “immigration” in their title, formal analysis and discussion, but “net immigration” in their illustrative projection, as the source of their data, and in the test of formulae from the analysis.

The point is no mere quibble. Net migration is the difference between immigration and emigration. Some of that emigration represents previous immigrants, but some occurs to members of the original population. The author use an assumption of 400,000 net immigrants in their illustration. Suppose, for sake of argument, that was the net consequence of 500,000 immigrants, 50,000 foreign-born emigrants and 50,000 native-born emigrants. In determining ultimate birth cohort size, the key question is the number of annual births occurring to immigrant women, once their full reproductive complement has been attained. The base for that subpopulation, in the example given, is the net flow of 450,000 rather than 400,000 women. Furthermore, one cannot determine the ultimate size of the native population by using ultimate birth cohort size as the radix of the conventional life table, because each native cohort is subject to attrition not only from mortality but also from native emigration. To repair these difficulties the requisite assumptions of the model are a fixed annual amount of immigration, and no emigration.

Such considerations may have led to Espenshade's subsequent recommendation (Espenshade, 1982) that emigration be incorporated in the stable model in rate form, i.e., treated as a kind of mortality, as outlined above.

In the same article, he asserted that a net migration rate as modification of mortality would be improper because the two components of the numerator, immigration and emigration, have different bases of exposure to risk. Emigration originates from within the population, whereas immigration originates in the rest of the world. This raises a serious question about the legitimacy of the rate model proposed above. The rest of the world provides

what may be considered the supply conditions for the immigration process. Yet where that supply is substantially in excess of the demand, the determination of the number (and characteristics) of immigrants is essentially a matter of the demand, i.e., of the immigration policy of the receiving country. Of the many considerations going into formulation of that policy, substantial weight would be given to the implications for the receiving population, at least in the form of the number of immigrants relative to the size of the receiving population. A wise government will be sensitive to the (probably misguided) perception of immigrants as competitors with the existing population for jobs, as well as for housing, welfare and the like. An aggregate concept of exposure to risk, in this sense, may differ from the conventional individual-based orientation, but it is far from meaningless.

From a formal standpoint, the conclusion is that the amount model is a valid way of incorporating immigration in the stable population model, but that the appropriate way to incorporate emigration, *ceteris paribus*, would be as a modification of survival. The acceptability of a model based on fixed rates of net migration would require tolerance for an unorthodox interpretation of exposure to risk. In choosing between these alternatives, two considerations have tipped the balance in favour of the amount model. First, net emigration of the native-born, in the case of Canada, seems very small; it appears likely, on the basis of admittedly inadequate data, that the considerable majority of native-born emigrants return. If so, the amount model would provide a tolerable approximation to the current situation. Second, there is a pragmatic consideration, evident in the foregoing account. A model in which rates of net migration are fixed implies, with the passage of time, an older age distribution of migrants because, as is highly likely, the base population becomes older. As will be shown subsequently, there are deleterious consequences from an older age distribution of migrants.

Selection of the amount model as the way to incorporate net migration in the stable population model permits specification of any desired ultimate population size – because the annual number of migrants required to attain that size is readily determinable. For the projections to be discussed in the next section, we propose to compare the replacement fertility model (closed to migration but with $R_0=1$) with a replacement migration model (current R_0 and fixed yearly migration). For the former, ultimate population size for females was shown above to be 16.823 million. The associated value for males requires first the multiplication of ultimate female birth cohort size by the sex ratio at birth (1.054) and then multiplication by the male expectation of life at birth (75.07912). The result is 16.418 million males.

To find the amounts of annual female and male net migration which would ultimately yield the same total population size, female and male, as have been determined for the fertility replacement model, the basic formula employed is that presented previously: $T = M*EM + U \cdot e_0$, where $U = M*RM/(1-R_0)$. Given the values of expectation of life for females at migration, EM , and at birth, e_0 , and the reproductive value for females, RM , a value of M , for females, is

indicated for any specified value of T. In the present situation, the value of M is 80,777 (compared with the current number of 97,079 net female migrants).

The same equations are used to determine the number of male migrants required to achieve the same ultimate total for them as in the fertility replacement model. For males, the expectation of life at migration, a function of the current age distribution of male emigrants, is 47.4053; the expectation of life at birth for males, in the 1994 life tables, is 75.07912. The ultimate birth cohort size for males is the product of the ultimate birth cohort size for females, already determined, and the sex ratio at birth, 1.054. With these values, the requisite number of male migrants turns out to be 86,448 (compared with the current 86,961). For perspective, the numbers of net migrants used in the migration model are about ten percent less than the annual average in the past decade, but about ten percent more than the annual average in the decade preceding that.

Although the total numbers of female and male migrants in the migration replacement model differ from the values observed for Canada, 1994-95, the age distributions of those migrants are retained: they are the determinants of the values of the key parameters of reproductive value, for females, RM, and of expectation of life after migration, EM, for females and for males.

Section 3. Comparison of Three Population Projections.

In the foregoing, the ultimate outcomes of several population models have been considered. Yet the ultimate is a long way off; the realistic interest is in the more immediate future. To this end we have prepared projections for the first century beyond the outset, specifically the updated postcensal population estimate for July 1, 1995, based on the 1991 census, adjusted for net undercount.

Projections have been prepared for three models: (1) Fertility fixed at the current level, with no migration, to be called the subreplacement model; (2) Fertility raised immediately to the replacement level, with no migration, to be called the replacement fertility model; (3) Fertility fixed at the current level, with fixed amounts of migration, to be called the replacement migration model. In all three models, mortality is fixed at the 1994 level. The annual amounts of migration, in the third model, are calibrated to yield the same ultimate population size, female and male, as in the second model.

Most previous work in this genre has emphasized population size, e.g., Gesano, 1994. Our preference is to consider the consequent age distributions. (Although that subject has not been ignored by others, the applications have concerned ad hoc forecasts rather than stable models. Cf. Ahlburg and Vaupel, 1993; Basavarajappa et al., 1993; Espenshade, 1994.) The justification for this emphasis is that the import of the age distribution is almost self-evident, whereas there seems to be no sensible advice, within very broad limits, that a demographer can give concerning whether any particular future population size would be too large or not large enough.

All calculations have been carried out for five-year age groups and five-year time periods, separately by gender. The results reported here are restricted to twenty-year age groups and time periods, for the genders combined. Although this is a coarse age/time grid, it suffices for present purposes; moreover the results of interest do not depend on gender differences.

Common to all three projections is the survival history of those cohorts constituting the population at the outset. For the two models closed to migration, the remainder of the projection is based on the succession of new birth cohorts. Cohort size declines by about twenty percent per generation in the subreplacement model, but shows no change in the replacement fertility model, in an asymptotic sense. In the early stages of the projection, cohort sizes for both reflect common deviations for exponential trend, because of irregularities in the age distribution of the current population.

There are three parts to the migration model. The first is identical with the results for the subreplacement model. The second is the subpopulation of migrants, gradually built up to an equilibrium size and age distribution over the course of the first century. The third part consists of the descendants of migrants, i.e., births to migrants, births to the children of migrants, and so forth, in the form of staggered overlapping series. This is the component with a determinate ultimate sum, approached over the long term, as discussed in the preceding section.

Table 1 shows population size, and annual growth rates, for the three models. All show growth in the first twenty years, as a consequence of the substantial population momentum implicit in the initial age distribution. By the time sixty years have elapsed, the growth rates are approximately as predicated in the conventional stable model, for the two closed models. The migration replacement model is by then four million larger than the fertility replacement model, yet we know that ultimately they will have the same population size, of 33.241 million. That is the explanation for the negative growth rate in the migration replacement model over the final decades of the first century; the same characterizes the rest of the history of population size for that model.

To examine the pace of the approach of population size to its ultimate value, in the migration replacement model, it is most convenient to think of the time series of birth cohort size. It can be shown that the difference between birth cohort size at year t and ultimate birth cohort size tends to decline exponentially at rate r (the intrinsic rate of natural increase). The inference then is a very long, very slow decline in population size, at a declining amount.

Were there a substantially smaller number of annual migrants, relative to initial population size, the population in the model would show a long slow rise at a declining rate. Consider, for example, the limiting case in which there is no initial population – colonization of an empty land. The projection values for that case can be obtained by subtracting the entries for the subreplacement model from those for the migration replacement model. Over the first century, at twenty-year intervals, the values would be 0, 3,745, 7,891, 11,853, 15,237,

18,132 (in thousands) with the ultimate population size, as before, 33,241 thousand – because that value does not depend on initial population size. In

Table 1.
Three Projections for Canada
 Population Size, N (in thousands), Growth Rate,
 r (per thousand per annum)

Years from Outset	Subreplacement		Replacement Migration		Replacement Fertility	
	N	r	N	r	N	r
0	29,606		29,606		29,606	
		2.3		8.0		5.1
20	31,025		34,770		32,793	
		-2.8		3.4		1.5
40	29,315		37,206		33,802	
		-7.6		-0.2		-1.1
60	25,187		37,040		33,075	
		-8.0		-0.5		0.1
80	21,466		36,703		33,150	
		-8.1		-0.4		0.1
100	18,245		36,377		33,237	

summary, then, in the migration replacement model the long-term path of population size is convex and declining when annual migrants are few relative to initial population size, but is concave and rising when annual migrants are many relative to initial population size.

Our next task is to find ways to parametrize the age distribution. The most common index in the literature is the mean age. For the current Canadian population, the mean age is 36.14. For the three models, at equilibrium, the respective mean ages are 45.51 for the subreplacement model, 44.00 for the replacement migration model, and 40.92 for the replacement fertility model. We have chosen not to show this calculation for the projections for the first century because it does not seem evocative of the socioeconomic relevance of age distribution differences. In its stead, we have devised three measures.

The first index is the dependency ratio, expressing the relation between those in the nonworking ages – here approximated by N_0 - N_{20} and N_{60} , following the

subscript convention of the life table T – and those in the working ages, N_{20} – N_{60} . Drawing on other discussions of the topic (Ahlburg and Vaupel, 1993; Espenshade, 1994), we have chosen the following form for the index $[3(N_0-N_{20})+5N_{60}]/(4(N_{20}-N_{60}))$. The results are shown in the upper panel of Table 2.

The dependency ratio rises in all three models. This is a subject of much current concern in developed countries. One obvious cause of the problem is the long wave in fertility: abnormally low two generations ago and abnormally high one generation ago. That sequence implies a lower dependency ratio when the children of the depression become seniors while the postwar children are still in the working ages, and then a higher dependency ratio when those postwar children in their turn become seniors.

Not so generally recognized is the circumstance that the sequence of lower dependency followed by higher dependency would have occurred, albeit of small magnitude, even without the long wave in fertility, because it is the natural ending to the demographic transition. That transition, in brief, is a movement from high to low levels of mortality and fertility, with mortality decline preceding fertility decline. Over the course of such a transition, the characteristic pathway of the dependency ratio is upward during the era in which mortality but not yet fertility is declining, downward during the era of fertility decline, and finally upward again.

The terminal reversal is occasioned by two independent circumstances. One may think of the dependency ratio as an average of two ratios, of children to parents, and of grandparents to parents. The former can be expressed as the level of parental fertility (modified somewhat by comparative survival); the latter can be expressed as the inverse of the level of grandparental fertility (modified substantially by comparative survival). In abstraction from survival considerations, the sum of parental fertility and the inverse of grandparental fertility rises when fertility declines below replacement, or when it stops declining at whatever level. In addition, the survival element in the child/parent ratio can change very little, provided mortality is low, whereas the survival element in the grandparent/parent ratio may increase appreciably. The discrepancy in outcome occurs because only when mortality is not low, as in older ages, can mortality decline have much effect on comparative survival.

In combination, the processes of change in fertility and in survival guarantee a rise in the dependency ratio in the terminal phase of the demographic transition. In perspective, the rise is characterized less aptly as a shift to a high ultimate level than as a return to more usual values from a low penultimate level. The penultimate stage, which developed countries are now about to leave, is a demographically fortuitous transitory era in which there are fewer junior dependents (because parental fertility is low) simultaneously with fewer senior dependents (because grandparental fertility had been higher).

Table 2.
Age Distribution Indices,
Three Projections, Canada

Years from Onset	Subreplacement	Replacement Migration	Replacement Fertility
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Dependency Ratio

0	0.708	0.708	0.708
20	0.823	0.790	0.900
40	1.121	1.014	1.127
60	1.143	1.031	0.998
80	1.134	1.022	1.031
100	1.139	1.022	1.031
Ultimate	1.138	1.017	1.025

Percent of Seniors Very Old

0	16.8	16.8	16.8
20	17.3	17.0	17.3
40	21.5	20.2	21.5
60	25.4	23.4	25.4
80	26.5	24.1	22.2
100	25.4	23.4	22.6
Ultimate	25.7	23.4	23.2

Labour Force Birth Rate (per thousand)

0	26.0	26.0	26.0
20	21.4	27.1	23.9
40	21.9	28.7	26.5
60	21.9	28.1	25.5
80	21.7	27.8	25.5
100	21.8	27.8	25.5
Ultimate	21.8	28.2	25.6

Beyond these commonalities, the dependency ratios show meaningful differences. Comparing the subreplacement model with that for replacement fertility, the former begins with a lower dependency ratio because of its lower fertility but soon shifts to a higher value; this is a characteristic negative feature of a declining population. The dependency ratio for the replacement migration model varies erratically relative to that for the replacement fertility model but is on average lower during the first century and remains lower at equilibrium.

The dependency ratio is a crude measure of the "costliness" of the age distribution. One way to reduce that crudity is to provide more information about its numerator. Since senior dependency is a major concern, and since that varies strongly and directly with age, we have chosen, as a second index to calculate, the proportion of those 60 and older who are 80 and older (N_{80}/N_{60}). This index is reconsidered subsequently.

The values of this index for the three models are shown in the central panel of Table 2. In every model, the proportion of seniors who are very old increases markedly with time. Because of the form of the index, the discrepancy between the values for the subreplacement model and for the replacement fertility model does not appear until $t=80$, but ultimately the former shows its characteristic age. Over the first sixty years of the projection, this index is lower for the replacement migration model, but ultimately it is almost the same as that for replacement fertility.

The third index of the age distribution is the labor force birth rate. In a sense, the purpose of this measure is to refine somewhat the denominator of the dependency ratio. To explain: any population, as an aggregate of individual characteristics, can be transformed, in adaptation to the changing environment, in two general ways. The constituent individuals may by retraining acquire different characteristics, or individuals with one set of characteristics, as they retire, may be replaced by entrants with a different set of characteristics. The latter process, which we have called "demographic metabolism" (Ryder, 1975), has been much more important than retraining in the technological evolution of the labor force in developed societies.

Demographic metabolism is a mode of aggregate adaptation, as a supplement to and compensation for the limits of individual adaptability. Strictly speaking it should encompass not only the inflow of persons into the labor force but also the outflow through retirement or death. In terms of aggregate adaptability, the inflow is of prime importance because it permits the active exploitation of role allocation. (Retirement is non-random, but it tends to reflect the disposition of individuals more than the needs of organizations.) Moreover, adaptability is implicated not only in the hiring of new workers by existent organizations but also in the creation of new organizations – for which the retirement process is for the time irrelevant. Accordingly we have elected to focus on the rate of inflow into the working ages, here labeled the labor force birth rate. For the model closed to migration, this has been estimated as $(N_0 - N_{40})/[40 \cdot (N_{20} - N_{60})]$. For the migration replacement model, this is supplemented by the ratio of migrants of ages 20 through 60 to the working age population, $(N_{200} - N_{60})$.

The indices are shown in the lower panel of Table 2. The values are dramatically different among the models. The subreplacement model, as expected because it is a declining population, has a lower labor force birth rate than the replacement fertility model. The highest values are associated with the replacement migration model – throughout the first century and at equilibrium. Moreover, the net migration data used here tend to understate the magnitude of flow into the working ages. Setting aside the continual turnover of the subpopulation of nonpermanent residents, the migration replacement model has 109 thousand net migrants in the working ages, made up of 138 thousand immigrants and an outflow of 29 thousand emigrants.

To summarize the account of age distribution indices, we have proposed three measures of the age distribution, and compared their changes over the first century, for projections based on three models. Depending on the index in question, there is convergence on ultimate values within 40 to 80 years. The primary measure of dependency increases substantially from its current value in all three projections, but to a much higher level for the subreplacement model. The index refining the numerator of the dependency ratio – the proportion of seniors who are very old – also increases substantially from the outset, and again particularly for the subreplacement model. To refine the denominator of the dependency ratio we have proposed the consideration of adaptive capacity, as measured by the labor force birth rate. The comparative assessment in this case is once again to the disadvantage of the subreplacement model, but substantially to the advantage of the migration replacement model. Yet it is only fair to note that there may be substantial aggregate costs associated with the assimilation of foreign-born entrants into the work force.

Section 4. The Role of Migrant Age.

The outputs of the migration replacement model are conditional on the age distribution of net migrants, assumed in the foregoing to be fixed at the current relative values for Canada, 1994-95. Arthur and Espenshade (1988) have studied the relation between population size and migrant age. First they assumed that all migrants arrive at the same age. In that model there is a strong inverse relation between ultimate population size and migrant age, for a specified total number of migrants. The outcome is apparent in the formula provided above for the relation between ultimate population size (T) and the number of migrants (M).

$$T/M = EM + RM * [e_0/(1-R_0)]$$

With increase in age at entry, the expectation of life after migration, EM, declines, and so does the reproductive value of a (female) migrant, RM. (For simplicity, the account in this section is confined to females.)

As they recognized, that demonstration is unsatisfactory because the model has zero variance in the age distribution of migrants. Arthur and Espenshade met this objection by taking an observed age distribution of migrants, and producing

from it two alternatives, one somewhat older and the other somewhat younger. Then they projected the three alternatives: the outcome was the same, but with a much attenuated contrast.

To cope with the same problem, in a more general way, we propose to use a model in which migrants are uniformly distributed over the range of ages from 0 to $2a$, where a is the mean age at migration. To justify this choice on more substantial grounds than mere convenience, we note that the square of the coefficient of variation of a uniform distribution is $1/3$. The same calculation for the age distribution of net migrants for Canada, 1994-95, gives 0.34 for females and 0.36 for males.

To implement the formula for the relation between annual number of migrants and ultimate population size, in a uniform distribution model, the requirements are the average values of $EM(a)$, the expectation of life after migration, and $RM(a)$, the reproductive value of a female migrant, over the distribution from 0 to $2a$, for various values of the mean age a . Given the female life table and net reproduction function for Canada, 1994, this is a straightforward exercise.

In accordance with the presentation in the preceding section, we have calculated the number of migrants required to achieve a specified ultimate population size, as a function of mean age of migrant, over what seems to be a realistic range from $a=20$ to $a=40$. Since the absolute numbers would depend on the size selected, and since a comparative appraisal is appropriate, we have chosen to express each outcome as a proportional deviation from that for mean age 30. The results are shown in Table 3, the column labeled $J(a)$. Since the values are very close to linear, the result can be expressed simply as follows: a higher mean age by one year implies an additional three percent of migrants required to achieve any given ultimate population size.

The other column in Table 3, labeled $K(a)$, is the proportion of the ultimate population which is migrant. To obtain this, note that the migrant population is $M(a)*EM(a)$, and the nonmigrant population $M(a)*RM(a)*e_0/(1-R_0)$. It follows that $K(a)$ is the reciprocal of $([e_0/(1-R_0)]*[RM(a)/EM(a)] + 1)$. Note that the expression is independent of the number of migrants, $M(a)$. Also, since $[e_0/(1-R_0)]$ is fixed, $K(a)$ depends on the relative change in $RM(a)$ and $EM(a)$. Over the selected range, $RM(a)$ declines by about 50 percent and $EM(a)$ declines by about 30 percent; moreover, the ratio declines at a decreasing rate. Accordingly $K(a)$ rises at a decreasing rate, as shown in Table 3.

The proportion of the population migrant is an important element in calculations of parameters for the total population. The distributional properties of the nonmigrant population are fixed: they are characteristic of the stationary population in the life table. Accordingly, change in any parameter for the total population, as a consequence of variation in migration age, depends on change in the parameter in question, for the migrant population, and on change in the proportion of the population migrants.

Table 3.
Percent difference between number of migrants
of mean age a , and number of migrants of mean age 30,
to achieve a given population size, $J(a)$; and percent of the
ultimate population migrant, $K(a)$.

a	$J(a)$	$K(a)$
20.0	-29.4	21.6
22.5	-22.0	22.9
25.0	-14.7	24.1
27.5	-7.3	25.1
30.0	0.0	25.9
32.5	7.4	26.5
35.0	14.8	27.1
37.5	22.3	27.5
40.0	29.8	27.8

$K(a)$ is significant in another way. Net migration is the only way to prevent a population declining if fertility remains inadequate. The cost of that solution – on the assumption that it is regarded by the receiving population as not without cost – is relative to the proportion of the population migrant. But we have seen that that is independent of the magnitude of the annual migrant inflow. It is a function of the reproductive value of females, $RM(a)$, and the expectation of life after migrating, $EM(a)$, both dependent on the age distribution of migrants. The “cost” of migration, as a way to achieve replacement, can be reduced by lowering the mean age at entry of migrants.

As a footnote, it is assumed throughout that the fertility and mortality of migrants are the same as for the receiving population. It would be feasible to achieve a specified ultimate population size with a smaller ultimate proportion migrant, if the migrants had higher fertility and mortality than the receiving population. But this would introduce another dimension into the determination of the “cost” of migration. The more different the migrants are from the receiving population, the greater the “cost.” Higher fertility and mortality are reasonable representatives of such difference. In crude terms, this mode of population replacement would have lower “cost” because there were relatively fewer of “them,” but higher “cost” because “they” differed more from “us.”

The next parameter considered is the mean age of the population. (Cf. Espenshade, 1994) For migrants at any particular age x , there is a mean age for the (stationary) population they produce subsequent to arrival. (A simple way to calculate these mean ages from a single-year life table is to create a new life-

table column, say V_x , where V_x is to T_x as T_x is to L_x . Then the mean age for entrants at age x is $[(V_x/T_x)+(x-0.5)]$.

Table 4 provides, for alternative values of mean age of migrant, the consequent mean age of the migrant population, and of the total population. The latter is the weighted average of the mean age for the nonmigrant population, fixed at 41.91, and that for the migrant population, where the weights are to be inferred from column K(a) in Table 3. As the mean age at entry increases, the mean age of the migrant population becomes higher, and its weight in the total larger. Nevertheless, the increase in mean age of the total population averages only 0.11 year for each year of increase in mean age of migrant at entry.

Since it is difficult to perceive whether the observed change in mean age of the population is large or small in its consequences, we turn to an examination of the indices of the age distribution, described in the preceding section. These are shown in Table 5 for various mean ages of migrant at entry. The dependency ratio varies directly with the mean age of migrant, by about 0.6 percent per year of age. For perspective on these values, the dependency ratio for the equilibrium non-migrant population is 1.097, a value exceeded by that for the total population only when migrants have mean age of at least 30. It is clearly advantageous, from the standpoint of the dependency ratio, to seek a younger age distribution of migrants at entry.

Table 4.
Mean Age of Migrant Population and of Total Population,
as a function of mean age of migrants, a.

a	Mean Age of Population Migrant	Total
20.0	51.19	43.91
22.5	52.16	44.26
25.0	53.07	44.60
27.5	53.92	44.92
30.0	54.70	45.21
32.5	55.40	45.49
35.0	56.02	45.73
37.5	56.56	45.93
40.0	57.00	46.10

An interesting feature of the other two indices is that the proportion of seniors very old is invariant over the range of mean entry ages from 20 to 30, and the labor force birth rate is invariant over the range from 30 to 40. These outcomes are the algebraic consequences of the assumption of uniform distribution of migrants by age, in conjunction with the particular age limits used in the

formulae for the indices. That by no means trivializes the outcome. Provided the distribution of migrants by age does not depart substantially from uniform, the indices would show similar insensitivity to mean age of migrant at entry, over the indicated ranges.

A higher proportion of seniors very old, and a lower labor force birth rate, would be interpreted as unfavorable. The former is associated with a higher, and the latter with a lower age at entry. The variation is significant only at the extremes of entry age, and in any event represents a trade-off between these two characteristics of the age distribution.

Table 5.
Age Distribution Indices as a Function
of Mean Age of Migrant, a .

a	Dependency Ratio	Percent of Seniors Very Old	Labor Force Birth Rate (per thousand)
20.0	1.057	26.8	26.3
22.5	1.066	26.8	26.8
25.0	1.079	26.8	27.3
27.5	1.097	26.8	28.0
30.0	1.120	26.8	28.8
32.5	1.143	26.9	28.8
35.0	1.162	27.1	28.8
37.5	1.177	27.6	28.8
40.0	1.189	28.3	28.8

The decision to base the calculations in this section solely on the females requires some justification. To achieve the objective of a specified ultimate population size, with fertility and mortality fixed, the key parameter is the average reproductive value of a female, $RM(a)$. In principle it is feasible to reduce the proportion migrant in the population by reducing the number of male migrants, in the limit to zero. Yet female fertility would not remain fixed if there were a large increase in the female but not in the male population. The constraint on the supply of prospective parents in such a situation would become the size of the male population. Doubtless there would be some increase in birth output, but certainly far less than required to compensate for a substantial shortfall below replacement in reproductivity.

The implicit condition underlying the use of the net reproduction function for the receiving population as the basis for calculating a reproductive value for migrant females is that the supply of partners for the latter is as adequate as it is

for females in the receiving population. How to measure adequacy in this sense is a perplexing demographic problem.

The consequences for the above analysis of including males as well as females are small. For example, consider the proportion of the population migrant. When the average age of migrant female is 30, the value is 25.9 percent. With equal numbers of males and female migrants, of the same age, the value is 25.8 percent. There are two reasons why the value is a little less. For any migrant age, the proportion of total lifetime still to be spent after migration, where age at migration on average is 30, is 64 percent for females and 62 percent for males (because their respective survival curves differ). In the second place, the reproductive value of females, in terms of male births, is about five percent larger than it is in terms of female births, because of the sex ratio at birth. In consequence the native-born proportion of males is increased relative to that for females. In brief, there are differences, but they are insufficient to merit separate exposition.

Section 5. Qualifications

The migration replacement model described above is an imprecise instrument for policy guidance. In the first place, the data used (for distribution of migrants by age and gender) are the amounts of net migration in a particular year; those numbers have changed appreciably from year to year.

Net migration signifies all change in population size other than births and deaths. Although this is formally unproblematic, it does not suffice to inform immigration policy. In the 1994-95 Canadian data, four categories of population movement constitute net migration: 214,296 immigrants; 22,292 returning Canadian citizens; 45,862 emigrants; and a net decline of 7,315 in the number of nonpermanent residents (Statistics Canada, 1996). In the year considered, the last number was fortuitously trivial, but the same would not have been so in particular other years. In 1988-89, the subpopulation of nonpermanent residents increased by 141 thousand; in 1993-94, it decreased by 83 thousand. Policy with respect to non-permanent residents is a separate consideration from that concerning the permanent population. Moreover, the size of the former is difficult to estimate, in large part because of undocumented entries and exits. In retrospect, it would have been preferable to restrict attention to the permanent resident population.

The critical variable when one is considering immigration policy is the number of non-citizens taking up permanent residence in Canada. The other categories of net migration (returning Canadians, and emigrants) are uncounted; their numbers must be estimated from administrative data collected for other purposes. It would be preferable to model these types of population change as processes dependent on characteristics of the exposed population, as with mortality and fertility, because they are not subject to regulation. Lacking a base for returning Canadians, that category can only be handled in an ad hoc manner.

The migration replacement model is unsophisticated because the requisite information is missing, and must therefore be assumed away. With more information, a considerable advance could be made by distinguishing the native and foreign subpopulations, and evaluating separately their fertility, mortality and emigration. As an example of the importance of what is now necessarily missing, the key element in the calculation of ultimate birth cohort size, in the migration replacement model, is the ratio of the reproductive value of a foreign female migrant to the complement of the native net reproduction rate. Native and foreign net reproduction have been assumed identical, but that may not be so.

Furthermore, immigration policy is not determined solely in terms of its perceived consequences for the receiving population. Most immigrants are admitted by criteria independent of the achievement of demographic objectives. The 1976 Immigration Act established three classes of immigrant: the family class, admitted on the basis of family connection; the refugee class, admitted on humanitarian grounds; and a residual class admitted in terms of criteria like education and training, but also including kinship. In 1991, the former two classes were 60 percent of the whole (Basavarajappa et al., 1993). Since their levels are likely to change over time, there is substantial constraint on the feasibility of a demographically oriented immigration policy.

The next qualification to the relevance of the models is the assumption – in both the subreplacement and migration replacement models – that fertility remains fixed at its current level. Elsewhere (Ryder, 1990), for a comparable population in most respects, we have argued that future fertility is likely to remain close to its current value, with a higher likelihood of decline than rise, but the emphasis should be on the skepticism with which any fertility forecast should be regarded.

Future change in fertility would have large consequences for the outputs of the migration replacement model. Referring again to the key determinant of ultimate cohort size, the ratio $RM/(1-R_0)$, it is evidently sensitive to fertility change. To get an impression of the magnitudes involved, we assumed that fertility became ten percent higher or lower than currently and determined that the ultimate proportion of the population migrant would decline from 26 percent to 16 percent in the former case, and would rise to 35 percent in the latter case. (The elasticity of proportion of population migrants, with respect to fertility level, is approximately 3.6.) This would be important for the comparisons reported above because, as noted in the preceding section, all age distribution measures are affected by the proportion migrant. As a more general statement, both the subreplacement model and the migration replacement model converge on the fertility replacement model (in terms of age distribution) as the fertility level rises toward replacement, and conversely should it decline.

The final set of qualifications concerning the work presented above begins with the assumption, common to all three models, that mortality remains fixed at its current level. Mortality decline is a dominant characteristic of the twentieth century, and it has a major influence on the age distribution, particularly in its

upper reaches. Moreover, mortality in the older ages is likely to decline appreciably in the future. Such decline is likely to affect the age distribution indices in all three models in much the same way. This raises the question of the validity of the age distribution indices.

To document the decline in mortality, we have calculated, from official life tables (Nagnur, 1986; Statistics Canada, 1995) mortality rates by age and gender for 1940-42, 1965-67 and 1990-92. The fifty-year span is divided in two to monitor change in change. The data are shown in the upper panel of Table 6. Decline is substantial throughout, although greater in the younger than in the older ages, and greater for females than for males. Of particular note with respect to the likelihood of continuation, there is no sign of a slackening of decline from the first to the second 25 years.

In the lower panel of Table 6, we show the increase in stationary population size consequent upon these mortality declines. For the replacement fertility model these would represent changes in the ultimate age distribution. Here the changes are relatively much greater in the older than in the younger ages. There are two simple reasons for this. First, the elasticity of change in p_x with respect to m_x is approximately $-m_x$. In other words, only when mortality is high does a change in mortality lead to much of a change in survival. Second, gains in survival cumulate age by age, so that the proportional increase necessarily enlarges with advance in age.

In reflection of the changes in stationary population size by age, the age distribution indices for the 1990-92 life table, in comparison with those for the 1940-42 life table, show an increase of 18 percent in the dependency ratio, an increase of 78 percent in the proportion of seniors very old, and a decrease of 4 percent in the labor force birth rate. These are the kinds of change to be anticipated henceforth, should mortality continue to decline.

The reshaping of the age distribution of replacement populations, particularly in the older ages, however measured, has been the subject of concern if not alarm. Some reconsideration seems in order. The index which changes most dramatically is the proportion of seniors who are at least age 80. The intent of this index is to measure the extent of frailty in the senior population. Based on the 1940-42 life table, the value of the measure was 13 percent; for the 1990-92 life table it was 23 percent. The assumption implicit in the comparison is that the same chronological age (80) has the same signification with respect to frailty, at the two time points, and thus independent of the change in survival between them.

When the chronological age is used to indicate frailty, the referent is length of time elapsed since birth. An alternative, more appropriate, referent in the circumstance would seem to be the length of time that will elapse until death. (Cf. Ryder, 1975). Accordingly, as an alternative measure, we suggest the proportion of the senior population who die within some specified arbitrary number of years, say five. In life table terms, the calculation is $1 - T_{65}/T_{60}$. That gives 28.2 percent for 1940-42 and 22.4 percent for 1990-92.

Table 6.
Change Over Time in Two Life Table Parameters

percent decrease in mortality ${}_nM_x$ from 1940-42 to 1965-67
and from 1965-67 to 1990-92

	Female		Male	
	1st 25 years	2nd 25 years	1st 25 years	2nd 25 years
0/20	64	66	60	64
20/40	68	37	37	27
40/60	41	35	12	40
60/80	34	34	8	29
80+	23	30	15	18

Percent increase in stationary population size ${}_nL_x$

	Female		Male	
	1st 25 years	2nd 25 years	1st 25 years	2nd 25 years
0/20	64	66	60	64
20/40	68	37	37	27
40/60	41	35	12	40
60/80	34	34	8	29
80+	23	30	15	18

Thus the index used in the body of the paper shows a rise (of 78 percent) whereas the new index, designed for the same purpose, shows a decline (of 21 percent). It is not surprising that the change is in the opposite direction. Since the particular cutting points are arbitrary, consider the subpopulation above age x . The proportion of them who are above age y is T_y/T_x ; that is the form of the original measure. But the proportion of them who die within $(y-x)$ years, the form of the newly proposed measure, is the complement of this, $1 - T_y/T_x$. No wonder they move in opposite directions. If the case for revision is valid, then the measure used in the above text is not only ill-suited, it is perverse.

The question of the appropriateness of using measures based on fixed chronological ages for comparisons over time is quite general. Consider, for example, the age limits of the dependency ratio. Age at entry is likely to rise

over time as the demands for education increase with technological advance. The question of the retirement age is more complex. To the extent that it is justified as a physiological correlate, it requires reconsideration as mortality declines. It is partly dependent on custom and law, and they may change over time. To some extent it reflects personal choice, but to some extent the choice of the employer, based on evidence or assumptions concerning the decline of productivity with age. The issue is not the particular upper age limit selected for calculating the index, but retaining the same limit over time.

A comparable issue concerns the equal treatment by gender in calculating the dependency ratio. If that ratio is used to indicate the taxes required from workers to support those in the dependent ages, and if the entry of women into the labor force is strongly impeded by social practice, a measure which fails to take gender into account is faulty. Relaxation of those impediments over time would realistically reduce the ratio of dependents to workers, but that would not be seen from any age distribution index in which the role of gender is held fixed over time.

The general point is that the measures of age distribution reported in the main body of the text, because they implicitly assume the invariant substantive significance of age and gender, are suspect during a time of mortality change and more broadly of social change. Indeed it is not unlikely that large changes in indices measured with fixed boundaries by age and gender will provoke not only responses in the form of immigration policy but also responses directed at the boundaries themselves. Status ascription by gender has been and will continue to be an important institutional variable in projections; status ascription by age is likely to become a comparably important institutional variable. In consequence, exercises like the present one can provide little more than a suggestive skeleton of what is required for meaningful analysis.

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