ZENO PARADOX: A RELATIVISTIC APPROACH TO SOLUTION

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Abstract

Zeno of Elea was brilliant producing paradox [1]; the most famous is the story of Achilles and the tortoise. It can be summarized in these words: Achilles and the tortoise decide to have a race. Because Achilles can run twice as fast as the tortoise he gives her a long head start. Now, says Zeno, by the time Achilles reaches the tortoise's starting point she would have moved ahead by half the distance of her lead. And by the time Achilles reaches that point she would have moved on by half of that distance. And so on, and so forth, ad infinitum. Achilles is never able to catch up with the tortoise, because at each point, by the time he has covered the distance between them, she will always have moved on further by half of that distance. As Magee [1] points, it is here an impeccable logical argument that leads to a false conclusion. As Borges [2] reports, many previous works had focused looking for a fault in the logic [3] but they all have failed, so Borges suggests looking back to the concept of our world.

In this work, we shall use such suggestion focusing in the relativity theory. We are convinced that the inaccessibility of the paradox lies in work under the Galilean transforms in the Newtonian world. Despite this, we will try to see the problem under the Lorentzian transforms in the Eistenian world. For this purpose we first take a look at the paradox in Galilean mathematical terms and then we will look at the paradox under Lorentzian transform.

Introduction

The Zeno paradox has an interest, especially in quantum mechanics where had been already treated as a singular effect named the quantum Zeno effect [4,5]. In the present work, we will continue with the Malykin [6] discussion and also to present the paradox in maths words in order to shed light to it.

The Zeno paradox could be written in mathematical terms by the use of Galilean transforms. So we can write the all time position of Achilles like this:

$$X_A = V_A t \qquad (1)$$

and the all time position of the tortoise:

$$X_T = X_0 + V_A t \qquad (2)$$

where V_A and V_T are the Achilles and tortoise respecting velocities, X_0 is the initial tortoise position and t is the time. As in the introduction example, $V_A = 2\ V_T$, and also, must exist a time t_0 so that $V_A \cdot t_0 = X_0$. Hence, $t_0 = X_0/2V_T$. Then, with the help of easy operations, we can rewrite the equations (1) and (2) at the time that Achilles arrives to X_0.

Achilles: $$X_{A1} = X_0 = V_A t_0$$

And the position of the tortoise at the same time:

$$X_{T1} = X_0 + V_T t_0 = X_0 + \frac{V_T X_0}{2V_T} = X_0 + \frac{X_0}{2}$$

Then Achilles needs a distance equal to $V_A \cdot t/2 = X_0/2$ to reach to the tortoise. It means, that distance is $2V_T \cdot t_0/2$, hence $t_0/2 = X_0/(4V_T)$

And the new positions will be:

Achilles: $$X_{A2} = X_0 + \frac{X_0}{2}$$

And tortoise:

$$X_{T2} = X_0 + \frac{X_0}{2} + V_T \frac{t_0}{2} = X_0 + \frac{X_0}{2} + \frac{X_0}{4}$$

And so on. If we carry on with this procedure we find two similar series. Already founded by Scottish mathematician James Gregory [7, 8], who has demonstrated and discussed the convergence of this series. From now on, we will name the Achilles position and the Tortoise position as X_{AN} and X_{TN}, to illustrate those series behaviour.

$$X_{AN} = X_0 \sum_0^N \left(\frac{1}{2}\right)^N \qquad (3)$$

$$X_{TN} = X_0 \sum_0^N \left(\frac{1}{2}\right)^N = X_0 \sum_0^N \left(\frac{1}{2}\right)^{N+1} \qquad (4)$$

Mathematically, the paradox lies in asking both series to be equal for some N value. Because of philosophical reasons, the N number can not be infinite. To take this inappropriate number (∞) is to think that Achilles already reaches the tortoise and this is forbidden in the logical steps. It is clear, that for any N number different from infinity, both series cannot be equal.

To make things easier we can rewrite the equations (3) and (4) as successions.

Remember that $\sum q^M = q(1-q^M)/(1-q)$, where q is the reason term of the series. In this example q=½.

We are asking to

$$X_0 \frac{\left[1-\left(\frac{1}{2}\right)^{N+1}\right]}{1-\left(\frac{1}{2}\right)} \text{ be equal to } X_0 \frac{\left[1-\left(\frac{1}{2}\right)^{N}\right]}{1-\left(\frac{1}{2}\right)} \quad (5)$$

for some *N* value.

This is clearly inconsistent. So we will take the Borges suggestion.

Results and discussion

In the Eistenian world we have to think again in the concept of simultaneity. What does it mean to be at the same time, at the same place? Both Achilles and the tortoise will not have any longer the same time, unless there exists an universal privileges clock. So, we must focus in only one of the subjects of our example. Let us think Achilles' situation under Lorentzian transform. The new first distance between Achilles and the tortoise is not just X_0, but X_0/γ. Where γ is $\sqrt{1-[V_A/C]^2}$; where *C* is the speed of light in the vacuum and Achilles' self time will be t/γ.

Hence, the hypothesis, the conditional equation (5) will turn to:

$$\frac{X_0}{\gamma} \frac{\left[1-\left(\frac{1}{2}\right)^{N+1}\right]}{1-\left(\frac{1}{2}\right)} = X_0 \frac{\left[1-\left(\frac{1}{2}\right)^{N}\right]}{1-\left(\frac{1}{2}\right)} \quad (6)$$

Now, with the aid of the γ term (which, in general, is very close to 1 for no relativistic movements) it cannot be said that it will not exist any *N* value different from infinity which will allow thinking that equation (6) is true. From eq. (6) it is easy to find that we have to ask to the *N* number to respect:

$$\frac{\left[1-\left(\frac{1}{2}\right)^{N+1}\right]}{1-\left(\frac{1}{2}\right)^{N}} = \gamma$$

Of course, the discussion is not finished. Despite we worked with an example, it is not hard to be more general with velocity relations. Even more, the paradox must be looked under other referencing systems or more complex theories. The Heisenberg uncertainty principle can give many views, taking into account that we are working with almost infinitesimal distances.

From the philosophical point of view, we think that this treatment can shed light on a full theory of the world. If the Zeno paradox cannot be resolved under a simple theory of the world (like the Greeks' or Newtonian), does it mean that the more complex theory, like relativity, is not just true but also necessary?

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