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Sanskrit Recension of Persian Astronomy: The Computation of True Declination in Nityānanda's *Sarvasiddhāntarāja*

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CONTENTS

HISTORY OF SCIENCE IN SOUTH ASIA 10 (2022) 68–168
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LIST OF TABLES

1 INTRODUCTION

In a recent publication, I discussed how Nityānanda Miśra, a seventeenth-century Sanskrit astronomer at the court of the Mughal emperor Shāh Jahān (1592–1666), translated Mullā Farīd al-Dīn Dihlavī's Indo-Persian *Zīj-i Shāh Jahānī* (c. 1629/30) into a Sanskrit table-text[1] (Misra 2021). Nityānanda's *Siddhāntasindhu* 'Ocean of *siddhānta*s' (c. early 1630) is an example of how a Persian *zīj* (a handbook of astronomical tables) is rendered into Sanskrit through a complex translation project that bridges the domains of sociocultural history and scientific innovation. My paper focused on the linguistic (syntactic, semantic, and communicative) aspects in Nityānanda's Sanskrit translation of Mullā Farīd's Persian text; in particular, on the computation of the *true* declination of a celestial object commonly discussed in the sixth chapter of the second discourse (*maqāla-i duvum*) of the *Zīj-i Shāh Jahānī* and the second part (*dvitīya-kāṇḍa*) of the *Siddhāntasindhu*.

Historically, the *Siddhāntasindhu* is Nityānanda's first attempt at presenting Islamicate astronomy to his fellow Sanskrit astronomers.[2] The *Siddhāntasindhu* retains the structure of the *Zīj-i Shāh Jahānī* in presenting the translated contents; however, in several instances, it groups topics under traditional Sanskrit categories. For example, at the end of his *Siddhāntasindhu* Part II, Nityānanda subsumes the topics discussed in its twenty chapters as those that are *tripraśna-pracura-ukti-yukti-sahita* 'accompanied by many statements and rationales on the *tripraśna*' (Misra 2021: 84). The *tripraśna* (lit. three questions) is often a separate topic (*adhikāra*) in a Sanskrit *siddhānta* that discusses mathematical methods to determine the direction, place, and time;[3] it is not, however, a separate comprehensive category in Islamicate *zīj*es like the *Zīj-i Shāh Jahānī*. By invoking this familiar *siddhāntic* topic of *tripraśna*, Nityānanda attempts to situate, explicate, and appropriate foreign astronomical methods in his Sanskrit text more naturally.

The *Sarvasiddhāntarāja* 'The King of all *siddhānta*s' (1638) is Nityānanda's second attempt to adapt Islamicate ideas to the paradigms of Sanskrit astro-

1 Montelle and Plofker (2018) provide an in-depth study on the emergence, development, and influence of the genre of astronomical table-texts (*sāraṇī*s or *koṣṭhaka*s) in Sanskrit *jyotiḥśāstra*. In particular, see § 6.12 for an example of how Nityānanda transforms the presentation of tables in a Persian *zīj* to suit the format of a Sanskrit *koṣṭhaka* in his *Siddhāntasindhu* (pp. 245–248).

2 The word Islamicate simply refers to the cultural outputs (artistic, literary, and scientific works) of the Arabic and Persian language traditions. As Hodgson originally proposed, "'Islamicate' would refer not directly to the religion, Islam, itself, but to the social and cultural complex historically associated with Islam and the Muslims, both among Muslims themselves and even when found among non-Muslims" (Hodgson 1974: 59).

3 In Sanskrit astral sciences (*jyotiḥśāstra*), a *siddhānta* is a comprehensive canonical treatise that includes, *inter alia*, discussions on planetary computations, astronomical parameters, and spherical geometry.

nomy.[4] Unlike the *Siddhāntasindhu*, the *Sarvasiddhāntarāja* follows the material and the metrical standards of a traditional Sanskrit *siddhānta*. In it, Nityānanda demonstrates his originality, innovation, and proficiency in integrating Greco-Islamicate ideas (*yavana-mata*) with traditional Sanskrit canonical (*saiddhāntika*) and mythohistorical (*paurāṇika*) thought.[5]

The contents of the *Sarvasiddhāntarāja* are arranged in two main parts (*adhyāya*s): the *gaṇitādhyāya* 'chapter on computations' and the *golādhyāya* 'chapter on spheres'. The *gaṇitādhyāya* discusses various topics (*adhikāra*s) like philosophical rationales (*mīmāṃsā*); mean and true positions of planets (*madhyamagraha* and *sphuṭagraha*); three questions (*tripraśna*) of direction, place and time; solar and lunar eclipses (*sūryagrahaṇa* and *candragrahaṇa*); elevation of the lunar cusps (*śṛṅgonnati*); planetary and stellar conjunctions (*bhagrahayuti*); and planetary and stellar altitudes (*bhagrahāṇām unnatāṃśa*). And the *golādhyāya* includes discussions on the topics of cosmography (*bhuvanakośa*), the armillary sphere (*golabandha*), and astronomical instruments (*yantra*).[6] Together these two parts describe the astronomical parameters, mathematical procedures, and the underlying geometry that helps calculate the movement of celestial objects (planets, stars, etc.) in the sky.

Beyond a standard register of topics, the *gaṇitādhyāya* of the *Sarvasiddhāntarāja* also discusses the computation of 'true declination' (*spaṣṭa-krānti*) of a celestial object as a separate topic in the chapter, viz. the *spaṣṭakrāntyādhikāra* 'topic on true declination'. As § 2.1 describes, all seven extant (and complete) manuscripts of the *Sarvasiddhāntarāja* include the *spaṣṭakrāntyādhikāra* as a separate section in the *gaṇitādhyāya*.[7] In most Sanskrit *siddhānta*s, the mathematics of computing the true declination of a celestial object is embedded in the general discussions on related topics (e.g., in the *tripraśna*, *śṛṅgonnati*, *bhagrahayuti*, etc.); see, for example, Appendix A. The *Sarvasiddhāntarāja* is different from other *siddhāntic* texts in treat-

4 See Misra (2016: §§ 1.1 and 1.2 on pp. 1–20) for a discussion on Sanskrit astronomy in early-modern India, in particular, the contribution of Nityānanda and his *Sarvasiddhāntarāja*.

5 See Pingree (2003), Montelle, Ramasubramanian, and Dhammaloka (2016), Montelle and Ramasubramanian (2018), and Misra (2016) for recent studies on Nityānanda's *Sarvasiddhāntarāja*, and is connection with Islamicate astronomy.

6 Misra (2016: 18–32) describes the structure and content of Nityānanda's *Sarvasiddhāntarāja* in detail. The chapter on astronomical instruments (*yantrādhyāya*) is sometimes considered as a separate (third) chapter along with the chapters on computations (*gaṇitādhyāya*) and spheres (*golādhyāya*).

7 Pingree (1991: Table SR on p. 29), Misra (2016: Table 1.6 on p. 21), Montelle, Ramasubramanian, and Dhammaloka (2016: Table 1 on p. 2) and Montelle and Ramasubramanian (2018: Table 1 on p. 1) lists the topics in Nityānanda's *Sarvasiddhāntarāja*. However, the *spaṣṭakrāntyādhikāra* is not included in the list of topics of the *gaṇitādhyāya* or the *golādhyāya* of the *Sarvasiddhāntarāja* in any of these studies. It is only recently in 2019 that I discovered the *spaṣṭakrāntyādhikāra* as a separate self-contained section towards the end of the *gaṇitādhyāya* in all seven manuscripts of Nityānanda's *Sarvasiddhāntarāja*.

ing the computation of true declination as a separate *adhikāra* in the *gaṇitādhyāya*. The metrical verses in the *spaṣṭakrāntyādhikāra* of Nityānanda's *Sarvasiddhāntarāja* are taken from his *Siddhāntasindhu*, which itself is a Sanskrit translation of Mullā Farīd's Indo-Persian *Zīj-i Shāh Jahānī* (more on this in § 1.1). The attribution of Islamicate astronomy in the *Siddhāntasindhu* is conspicuous—the *Siddhāntasindhu* is self-admittedly a Sanskritised version of the *Zīj-i Shāh Jahānī*. In contrast, the origins of topics in the *Sarvasiddhāntarāja* remain veiled behind a complex narrative that syncretises Islamicate and Sanskrit astronomical ideas prevalent in seventeenth century Mughal India.

In the present study, I edit, translate, and analyse the contents of the *spaṣṭakrāntyādhikāra* in the *gaṇitādhyāya* of Nityānanda's *Sarvasiddhāntarāja* (henceforth identified as *Sarvasiddhāntarāja* I.*spa·krā*). My edition of the text is based on seven complete manuscripts of the *Sarvasiddhāntarāja* that were available to me. § 2 describes the manuscripts and my editorial conventions. The aim of this study is to understand the mathematics of the three methods of computing the true declination described in the text. In my technical analyses of these methods (in § 4), I include brief discussions on the history of these methods in other Islamicate and Sanskrit works, as well as the linguistic and mathematical peculiarities in Nityānanda's recension of these methods in the *Sarvasiddhāntarāja* I.*spa·krā*.

1.1 FROM THE *SIDDHĀNTASINDHU* TO THE *SARVASIDDHĀNTARĀJA*

Nityānanda's *Siddhāntasindhu* Part II.6 (*dvitīya-kāṇḍa*, *ṣaṣṭhādhyāya*) includes twelve metrical verses and four prose passages describing three methods to compute the true declination. These verses and passages are Sanskrit translations of corresponding Persian passages from Mullā Farīd's *Zīj-i Shāh Jahānī* Discourse II.6 (*maqāla-i duvum*, *bāb sheshom*); see Misra (2021: pp. 85–98). In the *Sarvasiddhāntarāja* I.*spa·krā*, Nityānanda excludes the prose passages but includes the twelve metrical verses, repeated almost verbatim, from the *Siddhāntasindhu* Part II.6. In addition, he adds two final verses (a penultimate verse and a terminal colophon) that are not found in the *Siddhāntasindhu* Part II.6. Table 1 lists the metrical verses and prose passages in Nityānanda's *Siddhāntasindhu* Part II.6 vis-à-vis the metrical verses in his *Sarvasiddhāntarāja* I.*spa·krā*.

1.1.1 *Choice of Sanskrit meters*

The fourteen verses in Nityānanda's *Sarvasiddhāntarāja* I.*spa·krā* are composed in an assortment of meters. Table 2 lists the verses and the names of their respective meters. All verses taken from the *Siddhāntasindhu* Part II.6 (see Table 1) are repeated in the same meter, even as there are minor grammatical changes in the text (more on this in § 1.1.2).

Passage	*Siddhāntasindhu* Part II.6 (*c. early 1630*)	*Sarvasiddhāntarāja* I.spa·krā (*1638*)
खगस्य०...०स्वदिक् ॥	[1] verse	verse 1
स्फुटाप०...०भवेत् ॥	[2] verse	verse 2
परम०...०ज्यका ॥	[3] verse	verse 3
किंवा०...०स्यात् ॥	[4] prose	–
अथ०...०कुर्यात् ॥	[5] prose	–
अथ०...०दिग्भवेत् ॥	[6] prose	–
यदि०...०र्भवति ॥	[7] prose	–
अथ प्रकारान्तरेण ॥ कदम्ब०...०द्गोलवित् ॥	[α] verse	verse 4
विषव०...०संप्रति ॥	[β] verse	verse 5[a]
भवन०...०कल्पिते ॥	[γ] verse	verse 6
विषव०...०दृक्कोटिः ॥	[δ] verse	verse 7[b]
खगस्य०...०सिञ्जिनी ॥	[8] verse	verse 8
तद्धनु०...०दोर्ज्यया ॥ तद्धनुः०...०दिक्तया ॥ स ग्रहस्य०...०शोधितः ॥	[9] verse–[10] verse	verse 9–verse 11
परस्फुट०...०दिक्समा ॥	[11] verse	verse 12
अन्यैर्यो०...०संवीक्ष्यताम् ॥	–	verse 13
इत्येत०...०पूर्तिम् ॥	–	colophon verse (on p. 98)

[a] verse 5 has the variant reading विषुव०...०तत्क्षणे.
[b] verse 7 has the variant reading विषुव०...०दृक्कोटिः.

Table 1: Comparison of the text in Nityānanda's *Siddhāntasindhu* Part II.6 and the *Sarvasiddhāntarāja* I.*spa·krā*. The text of the *Siddhāntasindhu* Part II.6 is edited from MS 4962 of the Khasmohor collection held at the City Palace Library in Jaipur, ff. 20r: 16 to 20v: 12. See Misra (2021: pp. 91–98) for the numbering of the metrical verses and prose passages in the *Siddhāntasindhu* Part II.6, and § 3 (pp. 94–98) for the numbering of the metrical verses in the *Sarvasiddhāntarāja* I.*spa·krā*.

Number	Verse	Name of the meter
1	खगस्य०...०स्वदिक् ॥	*vaṃśasthavila* (12 syllables/*pāda*)
2	स्फुटाप०...०भवेत् ॥	*pramāṇikā* (8 syllables/*pāda*)
3	परम०...०ज्यका ॥	*anuṣṭubh* (8 syllables/*pāda*)
4	कदम्ब०...०द्गोलवित् ॥	*pṛthvī* (17 syllables/*pāda*)
5	विषुव०...०तत्क्षणे ॥	*drutavilambita* (12 syllables/*pāda*)
6	भवन०...०कल्पिते ॥	*drutavilambita* (12 syllables/*pāda*)
7	विषुव०...०द्दक्कोटिः ॥	*āryā* (moraic meter)
8	खगस्य०...०सिञ्जिनी ॥	*pramāṇikā* (8 syllables/*pāda*)
9	तद्धनु०...०दोर्ज्यया ॥	*rathoddhatā* (11 syllables/*pāda*)
10	तद्धनुः०...०दिक्तया ॥	*rathoddhatā* (11 syllables/*pāda*)
11	स ग्रहस्य०...०शोधितः ॥	*rathoddhatā* (11 syllables/*pāda*)
12	परस्फुट०...०दिक्समा ॥	*vaṃśasthavila* (12 syllables/*pāda*)
13	अन्यैर्यो०...०संवीक्ष्यताम् ॥	*śārdūlavikrīḍita* (19 syllables/*pāda*)
col.	इत्येत०...०पूर्तिम् ॥ (colophon on p. 98)	*śālinī* (11 syllables/*pāda*)

Table 2: List of metrical verses in Nityānanda's *Sarvasiddhāntarāja* I.*spa·krā* with the names of their corresponding meters.

1.1.2 *Variations in the reading of the text*

There are occasional variations in the reading of the text in Nityānanda's *Siddhāntasindhu* Part II.6 and *Sarvasiddhāntarāja* I.*spa·krā*. Most of these variations are minor grammatical changes; however, some variations suggest a conscious attempt to reform the language of the translated text in the *Siddhāntasindhu* to a simpler (and more standardised) version in the *Sarvasiddhāntarāja*. I discuss below some of the main variations between the language of the verses in these two texts. As indicated before (in Table 1), the numbering of the verses of the *Siddhāntasindhu* Part II.6 follows the edition in Misra (2021: pp. 91–98), while the numbering of the verses of the *Sarvasiddhāntarāja* I.*spa·krā* is described in § 3 (pp. 94–98).

Renaming technical terms In the *Sarvasiddhāntarāja* I.*spa·krā*, verse 1 (first *pāda*), Nityānanda calls the second declination of a celestial object *anyatama-apama*, whereas, in the *Siddhāntasindhu* Part II.6, $[1]_{\text{verse}}$ (first *pāda*), he uses the expression *anyatara-apama*. The words *anyatara* and *anyatama* can be understood as a choice between 'either one of two' or 'any one among many' respectively. However, the mathematical context of this verse strongly suggests that these words refer to the second declination (and not the first). Therefore, the words *anyatara* and *anyatama* may be thought of as the comparative and superlative degrees of the pronomial adjective *anya* 'other' respectively. In both texts, the 'more other' or 'most other' declination is simply a reference to the second or other declination (*dvitīyā-krānti* or *anya-apama*) of the celestial object.

Also, in the *Sarvasiddhāntarāja* I.*spa·krā*, verse 1 (third *pāda*), Nityānanda calls the arc/curve of a great circle as *aṅka*, a word ordinarily used to indicate a number, measure, or mark. This word appears in the context of a quantity called the 'curve of true declination' (discussed in § 4.2.1). In the *Siddhāntasindhu* Part II.6, [1] $_{\text{verse}}$ (third *pāda*), the word *aṅka* appears as *aṃśa* 'share', closely following the Persian word *ḥiṣṣat* 'share' in Mullā Farīd's *Zīj-i Shāh Jahānī* Discourse II.6, passage [1]. (See notes 1 and 2 on p. 107 of § 4.2.1.)

Clarifying mathematical statements Nityānanda revises the name of an arithmetic operation, rendered in the *Siddhāntasindhu* as a literal Sanskrit translation of a Persian expression, to a simpler (and more familiar) mathematical statement in the *Sarvasiddhāntarāja*. In Persian mathematics, the expression *muṇḥaṭṭ kardan* 'to make low' refers to the arithmetic operation of dividing a sexagesimal number by sixty.[8] In the *Siddhāntasindhu* Part II.6, [2] $_{\text{verse}}$ and [8] $_{\text{verse}}$ (third *pāda*s), Nityānanda translates this operation as *adharī-kṛtā* 'having been lowered' using the past passive form of the verb *adharī-*$\sqrt{}$*kṛ* 'to make low'. However, in the *Sarvasiddhāntarāja* I.*spa·krā*, verses 2 and 8 (third *pāda*s), he uses the compound *tribhajyakoddhṛtā* 'having been divided by the Radius (i.e., the *sinus totus*)'. Here, the past passive form of the verb *ut-*$\sqrt{}$*hṛ* 'to divide' takes the word *tribhajyakā* (lit. the *sinus totus*) as its instrument, suggesting that a sexagesimal quantity is to be divided by the Radius or *sinus totus*. This expression offers a clearer mathematical statement than the literal (and obscure) translation in the *Siddhāntasindhu*.

In two other instance, viz. verse 9 (fourth *pāda*) and verse 12 (second *pāda*) of the *Sarvasiddhāntarāja* I.*spa·krā*, Nityānanda retains the use of the word *adhara* (or equivalently, its verbal form *adharī-*$\sqrt{}$*kṛ*) to refer to the 'lowering' of a sexagesimal quantity. MS Bn.II parses this word, mid-verse, to explain the meaning of lowering a number. (See note 1 on p. 120 in § 4.3.2 and the remark on p. 134 in § 4.6.)

8 Sixty is the value of the Radius or *sinus totus* in most Islamicate texts. Nityānanda's *Siddhāntasindhu* and *Sarvasiddhāntarāja* adopt this value following Mullā Farīd's *Zīj-i Shāh Jahānī*.

Occasionally, Nityānanda replaces a word by a (near-)synonymous construction that is metrically indistinguishable. For example, the word *samprati* 'at the present moment' at the end of fourth *pāda* in the *Siddhāntasindhu* Part II.6, [β]$_{\text{verse}}$ is replaced by the word *tatkṣaṇe* 'at that very moment' at the end of the fourth *pāda* of *Sarvasiddhāntarāja* I.*spa·krā*, verse 5. Such alterations, although grammatically trivial, convey the mathematics perspicuously.

Rewording complex expressions In going from the *Siddhāntasindhu* to the *Sarvasiddhāntarāja*, Nityānanda rewords certain expressions for better clarity while preserving the meter of the verses. For example, in speaking about the 'circle that reaches (*upaiti*) the pair of ecliptic poles and the pair of celestial poles'; in other words, the solstitial colure, Nityānanda uses the expression *kadamba-viṣava-dhruva-dvayam upaiti* in the first *pāda* of [4]$_{\text{verse}}$ in the *Siddhāntasindhu* Part II.6. However, in the corresponding first *pāda* of verse 4 in the *Sarvasiddhāntarāja* I.*spa·krā*, he changes the expression to *kadamba-yugala-dhruva-dvayam upaiti*. The compound (*samāsa*) formed by the words *kadamba-yugala* 'pair of ecliptic poles' and *dhruva-dvaya* 'pair of celestial poles' in the *Sarvasiddhāntarāja* is clearer in eliciting the meaning of the expression than the one formed by joining the words *kadamba* 'ecliptic pole', *viṣ*[*u*]*va-dhruva* 'pole of the equinox', and *dvaya* 'both' in the *Siddhāntasindhu*.[9]

Adopting standardised spellings In some instances, Nityānanda rewords certain expressions by choosing a more conventional spelling. For example, the construction *jñeyaḥ sadṛgbhujo 'sau* 'that [arc] should be known as the congruent arc (*sadṛś-bhuja*)' in the *Siddhāntasindhu* Part II.6, [δ]$_{\text{verse}}$ (third *pāda*) is changed to *jñeyaḥ sadṛgbhujākhyo* '[that arc] should be known as the congruent arc (*sadṛś-bhujā*) by name' in the *Sarvasiddhāntarāja* I.*spa·krā*, verse 7 (third *pāda*). In Sanskrit astronomy, the words *bhuja* and *bhujā* both refer to the arm/side of a planar figure or an arc of a circle. However, when invoked in relation to its geometrical complement *koṭi*, *bhujā* is the more conventionally acknowledged technical term.

Using synonyms In the *Siddhāntasindhu* and the *Sarvasiddhāntarāja*, Nityānanda uses grammatical expressions derived from a common verbal root, or alternatively, those from synonymous verbal roots to express the same mathematical operation. For example, multiplication is indicated by the word *nihanyate* 'is multiplied' (present passive form of the verb *ni*-$\sqrt{han}$) in the *Siddhāntasindhu* Part II.6, [2]$_{\text{verse}}$ (third *pāda*) and *hata* 'having been multiplied' (past passive participle form of the verb $\sqrt{han}$) in the *Sarvasiddhāntarāja* I.*spa·krā*, verse 2 (third *pāda*). A little further into the

9 The word *viṣava* is a suspected vernacular corruption of *viṣuva*, see Misra (2021: footnote [v] on p. 93).

text, the same operation is called *hata* 'having been multiplied' in the *Siddhāntasindhu* Part II.6, [11]$_{\text{verse}}$ (first *pāda*) and *guṇa* 'multiplied' (a historically attested but grammatically anomalous derivative of the verb $\sqrt{}$*guṇ* 'to multiply') in the *Sarvasiddhāntarāja* I.*spa·krā*, verse 12 (first *pāda*).

1.2 THE EPISTEME OF THE *SARVASIDDHĀNTARĀJA*

By repeating the three Islamicate methods to calculate the true declination of a celestial object in his *Sarvasiddhāntarāja*, having first translated them into Sanskrit in his table-text the *Siddhāntasindhu*, Nityānanda demonstrates what he considers worthy of inclusion in a *siddhānta* in Sanskrit *jyotiḥśāstra*. In his preliminary reflections (*mīmāṃsā*) at the beginning of the *gaṇitādhyāya* of the *Sarvasiddhāntarāja* (I.1.21), Nityānanda brings the full force of his literary and poetic skills to extol the eminence of a *siddhānta* with a verse in the *śārdūlavikrīḍita* meter flaunting the figurative device *vinokti*, lit. speech with the word *vinā* 'without':

किं भावेन विना रसो रसकथालापं विना किं वचः
किं वाक्येन विना सुकेलिकुतुकं केलिं विना किं रतम् ॥
किं सौख्यं रतवर्जितं तनुभृतां सौख्यं विना किं जग-
त्तद्वज्ज्योतिषशास्त्रमेतदखिलं सिद्धान्तहीनं च किम् ॥ २१ ॥

kiṃ bhāvena vinā raso rasakathālāpaṃ vinā kiṃ vacaḥ
kiṃ vākyena vinā sukelikutukaṃ keliṃ vinā kiṃ ratam ||
kiṃ saukhyaṃ ratavarjitaṃ tanubhṛtāṃ saukhyaṃ vinā kiṃ jagat-
tadvaj jyotiṣaśāstram etad akhilaṃ siddhāntahīnaṃ ca kim || 21 ||

What is sentiment without emotion? What are words without a discourse of sentiment? What is a desire for sweet dalliance without sentences? What is pleasure without dalliance? What is happiness without pleasure for human beings? What is a world without happiness? And likewise, what is this entire science of the stars without *siddhānta*s? 21[10]

Beyond such rhetorical language, Nityānanda also provides a more practical epistemic standard by which a *siddhānta* is to be judged. For example, in the *Sarvasiddhāntarāja* I.1.10, he says

10 Nityānanda uses a series of technical words from Sanskrit aesthetics and dramaturgy in this verse: for example, *rasa* (flavour or sentiment in a work), *bhāva* (passion or emotion), *keli* (amorous sport or dalliance), *rata* (coital enjoyment or pleasure), etc. The use of these words in climactic ascension accentuates the rhetorical power of his statement.

सिद्धान्त इत्यनुगतार्थपदप्रयोगाद्यद्वस्तुसूक्ष्मतरमस्ति तदेवसिद्धम् ॥
नान्यच्च गोलगणितद्वययुक्तिहीनं किंवोपलब्धिरहितं सुधियेति चिन्त्यम् ॥ १० ॥

siddhānta ity anugatārthapadaprayogād
yad vastu sūkṣmataram asti tadeva siddham ||
nānyac ca golagaṇitadvayayuktihīnaṃ
kiṃvopalabdhirahitaṃ sudhiyeti cintyam || 10 ||

Because of the use of the word *siddhānta*, the [etymological] meaning of which is pertinent, it should be understood by a learned man that only that subject matter which is most precise is established, not anything else which is devoid of the rationales of both spherics and computations or deprived of perception. 10

There are various pieces of information in this verse:

1. the word (*pada*) *siddhānta*, lit. established end, conveys a meaning (*artha*) that is etymologically pertinent (*anugata*);
2. using the word *siddhānta* in (the title of) a work in Sanskrit *jyotiḥśāstra* brings this meaning to bear upon the content of the work;
3. the implication of this is that the subject-matter (*vastu*) of a *siddhāntic* text can include only those topics are the most precise (*sūkṣmatara*) and hence considered true or established (*siddha*); and
4. the standards of inclusion require that the topics (a) adhere to the rationales (*yukti*) of computations (*gaṇita*) and spherical geometry (*gola*) and (b) agree with perception (*upalabdhi*).[11]

Further along in the *Sarvasiddhāntarāja* I.1.13–15, Nityānanda tells us about the nature of the contents (*vastu*) he includes in his *siddhānta*, as well as the sources from where this material derives:

ततोऽल्पया प्रक्रियया महत्या किंवा यथार्थं वितनोमि वस्तु ॥
सूक्ष्मप्रकारं बहुयुक्तियुक्तं दृष्टिप्रतीतं खलु वासनाढ्यम् ॥ १३ ॥

11 The word *upalabdhi* is translated as 'observation' in a few astronomical texts. For example, Bhāskara II, in his auto-commentary *Vāsanābhāṣya* on his *Siddhāntaśiromaṇi* (1150), cites Caturveda Pṛthūdakasvāmī (a ninth century commentator on the *Brāhmasphuṭasiddhānta*) to reaffirm that 'conforming with observations (*upalabdhi*) is the only explanation for the procedure to compute the latitude of interior planets' (see Chaturvedi 1981: 402). However, the term *upalabdhi* can also be variously understood as perception, apprehension, observation, or cognition. In its polysemous use, *upalabdhi* can refer to what is self-evident or cognisant inasmuch as it could refer to actual observations (confirming predictions).

दृष्ट्वा रोमकसिद्धान्तं सौरं च ब्राह्मगुप्तकं ॥
पृथक्स्पष्टान्ग्रहाञ्ज्ञात्वा सिद्धान्तं निर्ममे स्फुटम् ॥ १४ ॥

रोमकोदितखचारचातुरी दृक्तुलां व्रजति सर्वथा सदा ॥
सौरतन्त्रमिह वेदविद्विदुर्जिष्णुजोक्तमपि युक्तयुक्तियुक् ॥ १५ ॥

tato'lpayā prakriyayā mahatyā kiṃvā yathārthaṃ vitanomi vastu ||
sūkṣmaprakāraṃ bahuyuktiyuktaṃ dṛṣṭipratītaṃ khalu vāsanāḍhyam || 13 ||

dṛṣṭvā romakasiddhāntaṃ sauraṃ ca brāhmaguptakaṃ ||
pṛthak spaṣṭān grahāñ jñātvā siddhāntaṃ nirmame sphuṭam || 14 ||

romakoditakhacāracāturī dṛktulāṃ vrajati sarvathā sadā ||
sauratantram iha vedavid vidur jiṣṇujoktam api yuktayuktiyuk || 15 ||

Therefore, with few or many methods, I suitably put forth the contents [that include] the subtlest procedures, accompanied by several rationales, indeed recognised by doctrines, [and] enriched with demonstrations. 13

Having consulted the *Romakasiddhānta*, the *Sūryasiddhānta*, and [the *siddhānta* of] Brahmagupta individually,[12] [and] having known the true [positions of the] planets, I composed the true *siddhānta*. 14

By his own admission, Nityānanda consulted the treatises of Sūrya, Brahmagupta, and Romaka to compose his own *siddhānta*. He validates the choice of his corpus by prescribing epistemic merits to each of these texts:[13]

- The *Sūryasiddhānta* is like the Vedas and hence it carries the probative force of verbal authority (*śabda*), making it a valid source of knowledge (*pramāṇa*).
- The *Brāhmasphuṭasiddhānta* contains several apposite rationales (*yukti*) of computation which, when logically applied, produce valid knowledge (*pramā*) by means of inference (*anumāna*). This makes the *Brāhmasphuṭasiddhānta* a highly established (*bahu-yuktiyukta*) text.

12 The continuative form *dṛṣṭvā* (of the verb $\sqrt{}$*dṛś*) refers to seeing or consulting something to acquire knowledge about an object. In other words, *dṛṣṭvā* may also be translated as 'having regarded' or 'having understood'. Compare the word *upalabdhi* in footnote 11 on p. 79.

13 See Narasimha (2007) for a discussion on the epistemology and language in Indian mathematical astronomy, particularly, in the works of Nīlakaṇṭha Somayāji (c. 1444–c. 1544) from the Kerala (Nila) school of astronomy and mathematics. Also see Narasimha (2012) for the role of *pramāṇa*, proof, and *yukti* in Indian sciences.

- The *Romakasiddhānta* contains methods that agree with perception; in other words, methods that are readily apprehended and result in computations that agree with observations. In Sanskrit epistemology, direct perception (*pratyakṣa*) is one of the fundamental means of valid knowledge, and accordingly, the *Romakasiddhānta* naturally acquires the alethic status of doctrine.

1.2.1 *Nityānanda's* Romakasiddhānta

By the early seventeenth century, the *Brāhmasphuṭasiddhānta* (628) of Brahmagupta and the 'modern' *Sūryasiddhānta* (c. 800) of anonymous authorship were well-known (and highly influential) treatises in northern India.[14]

However, the *Romakasiddhānta* appears in the annals of Sanskrit *siddhāntic* literature at different times through different agencies. For example,

- Lāṭadeva's recension of a *Romakasiddhānta* in Varāhamihira's *Pañcasiddhāntikā* (c. sixth century) and Bhāskara I's *Āryabhaṭīyabhāṣya* (629);
- Śrīṣeṇa's *Romakasiddhānta* criticised by Brahmagupta in his *Brāhmasphuṭasiddhānta* (628);
- a *Romakasiddhānta* (sixteenth century?) of Śrīṣavāyana set as a dialogue between Romakācārya and Dhūmraputra; and even
- a *Romaśasiddhānta* (unknown date, possibly c. sixteen century) that records the conversation between the celestial sage Nārada (or Lord Nārāyaṇa) and Vasiṣṭha Romaśamuni; see Dikshit (1890) and Pingree (1970–94: pp. 517a–519a in Volume A5).

In each of these instances, the nature and content of the text varies according to what the compiler, the critic, or the interlocutor needs it to be.

The identity of Romaka (or Romaśa), the eponymous author of *Romakasiddhānta* (or *Romaśasiddhānta*), is just as transitory as the work itself. Romaka (or Romaśa) has been variously and vaguely identified as Roman (*romaka*), Greek (*yavana*), Muslim (*mausula*), a foreigner (*mleccha*), a Vedic seer (*muni* or *ṛṣi*), and even a spiritual preceptor (*ācārya*).

According to Nityānanda, Romaka is both a seer (in the line of Indian seers or *ṛṣi*s to whom the gods reveal the sacred sciences) and also the sun god Sūrya himself.[15] The divine personages of Romaka render a revelatory (*śruti*) and doctrinal

14 The *Brāhmasphuṭasiddhānta* and the modern *Sūryasiddhānta* are identified as the foundational treatises of the Brāhmapakṣa and Saurapakṣa respectively; see Plofker (2009: 70–72) for a review of the different 'schools' (*pakṣa*s) in Sanskrit mathematical astronomy.

15 In his *Sarvasiddhāntarāja* I.1.2, Nityānanda identifies Romaka as a patriarchal sage (*ṛṣi*) in the lineage of Vasiṣṭha, Pulastya, Garga etc. A similar genealogy of Romaka also appears in Jñānarāja's *Siddhāntasundara* (1503, *bhuvanakośādhikāra*

(*dṛṣṭi*) authority to the *Romakasiddhānta*. At the same time, the lack of a fixed identity enables other content to be subsumed under the authorship of Romaka allonymously. This allows Nityānanda to first translate the computational methods from Mullā Farīd's *Zīj-i Shāh Jahānī* (a near-verbatim repetition of Ulugh Beg's *Zīj-i Ulugh Beg*) in his *Siddhāntasindhu*, and from there, include these methods in his *Sarvasiddhāntarāja* via the indiscriminate writings of Romaka. Nityānanda's *Romakasiddhānta* (a Roman *zīj*) contains the precise (*sūkṣmatara*) methods supported by computations (*gaṇita-yukta*), geometry (*gola-yukta*) and perception (*upalabdhi*) that then makes it verily canonical.

1.2.2 *From the gross to the subtle*

The methods to compute the true declination of a celestial object in the *Sarvasiddhāntarāja* are subject to the same epistemic criterion of precision or exactitude (*sūkṣmatā*) as is applied to various other types of computations described in the text. In fact, in the *Sarvasiddhāntarāja* I.*spa·krā*, verse 13, Nityānanda denounces all previous methods (stated by other authors in many different ways in their own *siddhānta*s) as being imprecise or inexact. The movement from the imprecise to the precise—from the *sthūla* 'gross' to the *sukṣma* 'subtle'—is the validation (*pramāṇa*) that establishes the truth (*yathārtha-siddhi*) of any new procedure.

Nityānanda's contemporary Munīśvara, in his auto-commentary to his *Siddhāntasārvabhauma* (1646), the *Āśayaprakāśinī* or *Siddhāntatattvārtha*, introduces verses 41–42 from the section on the conjunction of planets and stars (*bhagrahayuti*) with the statement

> पुर्वोक्तक्रान्त्यानयनस्यासंगतत्वात्प्रकारान्तरेण संगतं स्पष्टक्रान्त्यानयनं श्लोकाभ्यामाह---
>
> *purvoktakrāntyānayanasyāsaṃgatatvāt prakārāntareṇa saṃgataṃ spaṣṭakrāntyānayanaṃ ślokābhyām āha*—
>
> On account of the inappropriate nature of the previously-stated [rules of] calculating the declination, [the author] declared an apposite [rule for] calculating the true declination by another method in [the following] two verses—

For Munīśvara, the movement from the inexact to the exact—from the *asaṃgata* 'disunited' to the *saṃgata* 'united'—is the motivation to propose a different method of computing the true declination.

---

of the *golādhyāya*, verse 4ab). A little further in the *Sarvasiddhāntarāja* (I.1.16–18), Nityānanda invokes an older restoration myth to identify Romaka as the sun god Sūrya. Sūrya, afflicted by Brahmā's curse, was born among foreigners in the city of Romaka and was called Romaka. Eventually, when the curse was lifted, Sūrya was reinstated as the sun god whereupon he wrote the *Romakasiddhānta* (see Pingree 1996: 477–478).

Both authors use the adjectives *spaṣṭa* 'clear' or *sphuṭa* 'distinct' to qualify the declination of a celestial object as being 'true' or 'correct'. This stands in natural opposition to the implied antonyms *aspaṣṭa* 'unclear' or *asphuṭa* 'indistinct'. Hence, an improvement of clarity or distinction makes a procedure better suited to qualify as being both precise and proper.

Interestingly, in verse 13, Nityānanda uses the expression *sphuṭa-apakrama* to refer to the true declination of a celestial object. While the word *apakrama* is attested as a technical synonym of declination in Sanskrit astronomy (along with the more common terms *apama* or *krānti*), in its ordinary use, it carries the meaning of 'deviating from the regular order'. Thus, Nityānanda's statement, in verse 13ab, can be read as

> **अन्यैर्यो बहुभिः प्रकारनिचयैः प्रोक्तः स्फुटापक्रमः सत्स्थूलो... (१३[प्र,द्वि])**
>
> *anyair yo bahubhiḥ prakāranicayaiḥ proktaḥ sphuṭāpakramaḥ satsthūlo...* (13ab)
>
> What is declared by many others, with multitudes of methods, [as the] true declination, [that] is actually imprecise. ... (13ab)
> *or*
> What is declared by many others, with multitudes of methods [and] evidently deviating [from the truth], [that] is just imprecise. ... (13ab)

With the latter interpretation, it is obvious that Nityānanda's method is then meant to restore faithfully the regular order that takes the procedure from its gross statement to its subtle expression.

1.3 OVERVIEW OF THE *SPAṢṬAKRĀNTYĀDHIKĀRA*

In the *spaṣṭakrāntyādhikāra*, Nityānanda regards a celestial object as a planet (*graha*, *khaga*, etc.) or a star (*uḍu*, *bha*, etc.) that has a non-zero latitude (*bāṇa* or *vikṣepa*). In other words, the declination of this celestial object is different from the declination of the Sun that moves on the ecliptic (and hence, possesses no latitude). The method to compute the true declination (*spaṣṭa-krānti* or *sphuṭa-apama*) of such an object from its (known) latitude is essentially a question of coordinate conversion—the ecliptic coordinate of latitude β is converted to the equatorial coordinate of declination δ.

In most Sanskrit *siddhānta*s, the typical prescription to convert the latitude β of a celestial object to its declination δ (corresponding to its longitude λ) involves

1. calculating the declination of the object assuming it has no latitude, i.e., simply calculating $\delta(\lambda)$, and then
2. adding or subtracting a corrected form of the latitude, say β_{corr}, to or from this declination; in other words, $\delta(\lambda) \pm \beta_{\text{corr}}$.

The corrected latitude β_{corr} is a form of polar deviation (measured with respect to the celestial pole) derived from its ecliptic latitude β (measured with respect to the ecliptic pole). See Appendix A for a review of the different methods to compute the true declination of a planet or star in medieval Sanskrit texts.

Nityānanda's first two methods (in verses 2 and 3) are simple computational rules without any explanation or derivation. However, these rules do not transform the latitude to any other coordinate system. Instead, they compute the true declination using the 'first declination' (simply called *krānti*) and the 'second declination' (*dvitīyā-krānti* or *anya-apama*) of the celestial object. These quantities, along with other geometrical objects on the sphere, are defined and explained in § 4.1. Also, §§ 4.2.1 and 4.2.2 describe, in detail, two important geometrical arcs that are used in the first two methods. The derivations and historical testimonies of the first two methods can be seen in §§ 4.4 and 4.5 respectively. Lastly, Appendix B discusses the relation between the first and second declinations, while Appendix C describes the mathematical equivalence between the first two methods of true declination.

For the third method, Nityānanda systematically explains the various quantities that constitute the final expression (through verses 4–13). The structure of his exposition is as follows.

1. Name and define the different geometrical arcs in relation to the position of a celestial object on the sphere, namely,
   - the circle congruent to the ecliptic in verse 4, see footnote 25 in § 4.1;
   - the maximum true declination and the maximum latitude in verses 5-6, see § 4.2.3; and
   - the congruent arc and the congruent complementary arc in verse 7, see § 4.2.4.
2. Describe the rules for computing the congruent arc, the congruent complementary arc, the greatest declination (i.e., the obliquity of the ecliptic), and the maximum true declination—along with the correct way to interpret the arc of maximum true declination for values greater than ninety degrees—in verses 8–11, see §§ 4.3.1 and 4.3.2.
3. Express the third method of true declination in terms of the arc of the maximum true declination and the congruent arc in verse 12, and discuss the special case when the celestial object (understood as a star) is stationed at the ecliptic pole in verse 13, see § 4.6.

Remark

The *Tantrasaṅgraha* (1501) of Nīlakaṇṭha Somayāji (c. 1444–c. 1545) offers an exception to the typical prescription seen in most Sanskrit *siddhānta*s. Nīlakaṇṭha proposes two rules to compute the true declination of the Moon (see Ramasubramanian and Sriram 2011: §§ 6.3 and 6.4 on pp. 359–369). As Plofker (2002: 87–91) elaborates, both these rules determine the true lunar declination from its ecliptic coordinates in two completely innovative ways. While the first rule may resemble Ibn Yūnis's first method of declination from his *al-Zīj al-Kabīr al-Ḥākimī* (see King 1972: 39.1(a) on pp. 290–293), there is currently no substantive evidence to suggest a transmission of ideas between the Islamicate texts in circulation in northern India and the Sanskrit works of the southern Kerala school.[16] It is very likely that Nīlakaṇṭha's accomplishments were the product of his own ingenuity, which would be in keeping with the exemplary achievements of the students of Mādhava of Saṅgamagrāma. Indeed, Nīlakaṇṭha's second rule adds credit to this hypothesis by being an original among the writings of the Kerala school.[17] Later procedural texts, for example, the *Karaṇapaddhati* (c. 1600) of Putumana Somayāji that follow the *vākya*-system of Mādhava of Saṅgamagrāma to encode astronomical parameters as alphasyllabic strings or lexical phrases, repeat Nīlakaṇṭha's second rule to compute the true declination (see Pai et al. 2018).

1.4 BEYOND THE *SARVASIDDHĀNTARĀJA*

Among the earliest testimonies of Sanskrit authors engaging with Islamicate astronomy, we find a Sanskrit work on astrolabes, the *Yantrarāja* 'The King of instruments', composed by the Jain astronomer Mahendra Sūri at the court of Sulṭān Fīrūz Shāh Ṭughlāq of Delhi in 1370. The *Yantrarāja* is a summary of the theoretical and practical knowledge on constructing astrolabes, composed in metrical Sanskrit verses. As Mahendra Sūri states, it is *sudhāvat tatsārabhūtam* 'the nectar-like purified summary' of *bahuvidhā yantrāgamā yavanaiḥ* 'several treatises on instruments composed by the foreigners' (see Raikva 1936: *Yantrarāja* I.3). Malayendu Sūri, a student of Mahendra Sūri, wrote an extended commentary on the *Yantrarāja* in c. 1382.[18]

16 Pingree (1978: 319) speculates that there is an influence of Islamic lunar theory in the *Sphuṭanirṇaya* and *Rāśigolasphuṭānīti* of the Kerala astronomer Acyuta Piṣāratī (c. 1550–1621). His supposition is based on the presence of Islam (notably, the Arab mercantile societies) in the Malabar Coast of Southwestern India during the fifteenth and sixteenth centuries. However, without any material evidence to support this claim, Plofker's observation that "it is far from clear how such a hypothetical Islamic-Kerala transmission would have taken place across ethnic and cultural boundaries, with no obvious social channel of communication between Arab merchant society in the cosmopolitan port cities and the rural *illams* of the scholar-priests" (Plofker 2009: 252) remains the most judicious statement on this question of transmission.

17 See K. V. V. Sarma (1973) and Sriram (2008) for an overview of the contributions of the Kerala (Nila) School to Indian mathematics and astral sciences.

18 See S. R. Sarma (1999), Plofker (2000), and S. R. Sarma (2000; 2019: Appendix D1

Plofker examines Mahendra Sūri's calculation of the equatorial coordinates of a star, in particular, the computation of its true declination δ from its ecliptic coordinates (β, λ). Mahendra Sūri proposed two methods to compute the true declination in his *Yantrarāja*. His student-commentator Malayendu Sūri commented upon these methods and provided elaborate worked-examples (see Plofker 2000: 41–44). Mahendra Sūri's methods are Sanskritised presentations of earlier Islamicate methods, but as Plofker (2000: 38) notes, "the resulting procedures reveal a curious blend of misinterpretation and re-interpretation according to approximate methods". The final expression in both his methods differs from the exact Islamicate expression, possibly due to a combination of transmission errors, incorrect interpretations, and a general unfamiliarity with Islamicate astronomy.

The two methods of true-declination computation in Mahendra Sūri's *Yantrarāja* also appear (as the second and third method) in the *spaṣṭakrāntyādhikāra* of Nityānanda's *Sarvasiddhāntarāja*. However, Nityānanda's statements are identical to the exact Islamicate expression of these methods. In Misra (2021), I discussed how polyglot savants in seventeenth-century Mughal India acted as intermediaries between the Indian and Islamicate domains of knowledge. It is then reasonable to think that Nityānanda also benefited from the linguistic and technical expertise of the cosmopolitan Mughal court as he translated Persian astronomy into Sanskrit.[19]

Away from the Mughal capital of Shāhjahānābād, in the city of Kāśī, two of Nityānanda's contemporaries, Munīśvara Viśvarūpa (b. 1603) and Kamalākara (b. 1610), also discussed Islamicate astronomy in their *siddhānta*s. While their opinions on various Islamicate astronomical ideas differed, they both agreed on the utility of the trigonometry of the foreigners (*yavana*s).[20] The (Islamicate)

on pp. 4351–4396) for studies on the *Yantrarāja*. Also, Pingree (1991: 52–54) surveys the development of the genre of instrument-texts (*yantra*) in Sanskrit *jyotiḥśāstra*; while S. R. Sarma (2008; 2019) provide a comprehensive study of Indian astronomical instruments.

19 In a recent publication, Nair (2020) has studied how Sanskrit philosophical and religious texts were translated into Persian by collectives of Sanskrit and Persian scholars at the Mughal courts. Nair interprets philosophical translations as cross-cultural intellectual 'dialogues' expressed through the creation of novel vocabulary. This interpretation also extends to technical translations, particularly those that bridge language barriers in communicating foreign science. It is, hence, not all together surprising to read Pingree's observation that "[t]he *Sarvasiddhāntarāja*...provided Jagannātha [the author of *Samrāṭsiddhāntakaustubha* (1726)] with the Sanskrit words to describe Ulugh Beg's planetary models, and Jagannātha's successors with some of the terminology with which they wrote of European astronomy" (Pingree 2003: 283). Over a hundred years after Nityānanda, his words continued to echo in the dialogues between Sanskrit and Islamicate astronomy beyond the Mughal court.

20 See Minkowski (2014: 122-127) for the history of the familial rivalry between the Kāśī Brahmins Munīśvara (of the Devarāta *gotra*) and Kamalākara (of the Bhāradvāja *gotra*).

methods of true-declination computation in Munīśvara's *Siddhāntasārvabhauma* (1646) and Kamalākara's *Siddhāntatattvaviveka* (1658) are discussed in Appendices D and E respectively. They reveal a common method of calculation shared between the two texts: a method seen earlier in the *Sarvasiddhāntarāja* of Nityānanda.

Nityānanda's efforts, and those of his contemporaries, are situated in a time when Islamicate theories of astral sciences were actively debated by Sanskrit scholars of Mughal India.[21] In their writings, we find a diverse range of opinions and positions extending from traditionalism to pragmatism, from scepticism to certitude, and from reconciliation to polemics. By including Islamicate ideas in his *Sarvasiddhāntarāja*, Nityānanda reveals his proclivity for cogent ideas regardless of their origins. His ability to adapt and assimilate foreign knowledge to the linguistic, structural, and epistemic demands of a Sanskrit *siddhānta* is a remarkable feat of his scholarship. As ongoing and future studies bring other aspects of his works to light, we can build a better picture of the man and the milieu that helped shape his thoughts.

1.5 STRUCTURE OF THIS PAPER

The enumerated list below provides an overview of the contents of the different sections, appendices, and the technical glossary included in this paper.

- **§ 2** includes a description of the manuscripts, the orthographic standards, and the typographic conventions adopted in preparing the edition of Nityānanda's *Sarvasiddhāntarāja* I.*spa·krā*, as well as a description of the general format of the glossary.
- **§ 3** includes the edited text and corresponding English translation of the metrical verses in the *Sarvasiddhāntarāja* I.*spa·krā*.
- **§ 4** includes the technical analysis of the three methods of true declination described in the *Sarvasiddhāntarāja* I.*spa·krā*. The section begins by defining various geometrical objects on the celestial sphere (in § 4.1), along with preliminary definitions and computations of the constituent arcs (in §§ 4.2

21 For example, Nṛsiṃha Daivajña, in his *Vasanāvarttika* (c. 1621), a commentary on Bhāskara II's *Siddhāntaśiromaṇi*, discusses and disparages the opinions of the foreigners (*yavana-mata*) in several places (e.g., in his commentary to *Siddhāntaśiromaṇi*, *bhagaṇadhyāya*, verses 1–6). Also, Raṅganātha (fl. c. 1630/50), the brother of Kamalākara, writes his *Lohagolakhaṇḍana* 'Critique of the sphere of iron' accepting the claims of the Persians (*pārasīka*s) that the blue sky is, in fact, a crystalline sphere and not of a sphere of iron (*loha-gola*) as the orthodox opinion suggests. In response, Gadādhara (fl. c. 1650), Munīśvara's cousin, counters Raṅganātha's position in his *Lohagolasamarthana* 'Vindication of the sphere of iron'.

and 4.3 respectively). This is then followed by the derivations and historical testimonies of the three methods of true declination (discussed separately in §§ 4.4, 4.5, and 4.6).

The Appendices included in this paper describe

- the computation of true declination in medieval Sanskrit texts (Appendix A);
- Nityānanda's derivation of the second declination from the first, in the topic on three questions (*tripraśnādhikāra*) of his *Sarvasiddhāntarāja* (Appendix B);
- the equivalence between the first and second methods of declination in the *Sarvasiddhāntarāja* I.*spa·krā*.2–3 (Appendix C);
- Munīśvara's method of computing the true declination in his *Siddhāntasārvabhauma* (Appendix D); and
- Kamalākara's method of computing the true declination in his *Siddhāntatattvaviveka* (Appendix E).

A glossary of technical Sanskrit terms in Nityānanda's *Sarvasiddhāntarāja* I.*spa·krā*, accompanied by their corresponding English equivalents, is appended at the end the paper, beginning on p. 165.

2 SOURCES AND STRUCTURE OF THE CRITICAL EDITION

2.1 DESCRIPTION OF THE MANUSCRIPTS

I HAVE EDITED THE VERSES in Nityānanda's *Sarvasiddhāntarāja* I.*spa·krā* using seven manuscripts.[22] Printed or digital copies of the manuscripts were made available to me by the following institutions: (i) the Saraswati Bhawan Library of Sampurnanand Sanskrit Vishwavidyalaya in Varanasi (Benaras), (ii) the Bhandarkar Oriental Research Institute in Pune, (iii) the National Archives of Nepal in Kathmandu in conjunction with the Nepalese-German Manuscript Cataloguing Project maintained by Asia-Africa Institute of University of Hamburg, (iv) the Fergusson College Library in Pune, (v) the Rajasthan Oriental Research Institute in Jaipur, and (vi) the Scindia Oriental Institute at Vikram University in Ujjain.

Table 3 lists the sigla I have used to identify the seven manuscripts of Nityānanda's *Sarvasiddhāntarāja* in this study.

Siglum	*Manuscript*
Bn.I	Benares (1963) 35741 from Saraswati Bhawan Library, Varanasi.
Bn.II	Benares (1963) 37079 from Saraswati Bhawan Library, Varanasi.
Br	BORI 206 of A 1883/84 from Bhandarkar Oriental Research Institute, Pune.
Np	NAK 5.7255 from National Archives of Nepal, Kathmandu identical to NGMCP Microfilm Reel № B 354/15 from Nepalese-German Manuscript Cataloguing Project, Asia-Africa Institute of University of Hamburg.
Pm	Poona Mandlik Jyotisha 15/BL 368 from Fergusson College Library, Pune.
Rr	RORI (Alwar) 2619 from Rajasthan Oriental Research Institute, Alwar.
Sc	SOI 9409 from Scindia Oriental Institute, Ujjain.

Table 3: List of sigla of the manuscripts of Nityānanda's *Sarvasiddhāntarāja*.

The list below provides a brief description of the seven manuscripts identified in Table 3. A more comprehensive description of MSS Bn.I, Bn.II, Br, Np, and Rr can be found in Misra (2016: pp. 47–72 and pp. 77–86).

22 A complete list of the sixteen known extant manuscripts (including incomplete copies) of Nityānanda's *Sarvasiddhāntarāja* can be found in Pingree (1970–94: pp. 173–174 in Volume A3, p. 141 in Volume A4, and p. 184 in Volume A5).

MS Bn.I Saraswati Bhawan Library, Benares (1963) 35741
Copied in Saṃvat 1804 (= 1747 CE), 84 folia of size 10.5 × 4.7 cm, 12 lines per page (approximate), written in the Nāgarī script. The *spaṣṭakrāntyādhikāra* appears on ff. 62r–62v, beginning with *khagasya bāṇo*… on line 2 of f. 62r. The verses are numbered from 58 to 70 in a regular order, with the concluding verse of the chapter numbered 71 on line 9 of f. 62v.

MS Bn.II Saraswati Bhawan Library, Benares (1963) 37079
Copied in Saṃvat 1895 (= 1838 CE) and Saṃvat 1936 (= 1879 CE), 85 folia of size 10.3 × 6.8 cm, 13 lines per page (approximate), written in the Nāgarī script. The *spaṣṭakrāntyādhikāra* appears on ff. 62v–63v, beginning with *khagasya bāṇo*… on line 12 of f. 62v. The verses are numbered from 1 to 13 with the verse *anyairye…saṃvīkṣyatām* (verse 13 in the edition) omitted. Half-verses 4ab and 4cd in the edition appear as verse 4 and 5 in MS Bn.II respectively. The concluding verse of the chapter is numbered 14 on line 4 of f. 63v.

MS Br Bhandarakar Oriental Research Institute 206 of A, 1883–1884
Copied in Saṃvat 1941 (= 1884 CE), 47 folia, 14 lines per page, written in the Nāgarī script. The *spaṣṭakrāntyādhikāra* appears on ff. 34r–34v, beginning with *khagasya bāṇo*… on line 14 of f. 34r. Like MS Bn.II, the verses are numbered from 1 to 13 with the verse *anyairye…saṃvīkṣyatām* (verse 13 in the edition) omitted. Also, the half-verses 4ab and 4cd in the edition appear as verse 4 and 5 in MS Br respectively, and the concluding verse of the chapter is numbered 14 on line 11 of f. 34v.

MS Np National Archives Nepal, NAK 5.7255 (NGMCP Microfilm Reel № B 354/15)
Date of copying unknown, microfilmed on 9 October 1972 CE, 96 folia, 9 lines per page, written in the Nāgarī script. The *spaṣṭakrāntyādhikāra* appears on ff. 71v–72v, beginning with *khagasya bāṇo*… on line 6 of f. 71v. The verses are numbered from 1 to 13 in a regular order, with the concluding verse of the chapter numbered 9 on line 2 of f. 72v.

MS Pm Poona Mandlik Jyotisha 15/BL 368
Copied from a Jayapura manuscript, allegedly written in Saṃvat 1696 = 1639 CE (the date of composition), 54 folia of size 27.9 × 19.1 cm, 18 lines per page in two text blocks of nine lines each, written in the Nāgarī script. The *spaṣṭakrāntyādhikāra* appears on ff. 40r–40v, beginning with *khagasya bāṇo*… on line 16 of f. 40r. Like MSS Bn.II and Br, the verses are numbered from 1 to 13 with the verse *anyairye…saṃvīkṣyatām* (verse 13 in the edition) omitted. Moreover, the half-verses 4ab and 4cd in the edition also appear as verse 4 and 5 in MS Pm respectively, and the concluding verse of the chapter is numbered 14 on line 12 of f. 40v.

MS Rr Rajasthan Oriental Research Institute (Alwar) 2619

Copied on Thursday 10 *śuklapakṣa* of Kārttika in Saṃvat 1903 (= 29 October 1846 CE), 60 folia, 14 lines per page, written in the Nāgarī script. The *spaṣṭakrāntyādhikāra* appears on ff. 45r–45v, beginning with *khagasya bāṇo*... on line 13 of f. 45r. Like MSS Bn.II, Br, and Pm the verses are numbered from 1 to 13 with the verse *anyairye...saṃvīkṣyatām* (verse 13 in the edition) omitted. Like the other three manuscripts, the half-verses 4ab and 4cd in the edition appear as verse 4 and 5 in MS Rr respectively. The concluding verse of the chapter is numbered 14 on line 11 of f. 45v.

MS Sc Scindia Oriental Institute 9409

Date of copying unknown, 190 folia of size 10.5 × 4.5 cm, 7 lines per page, written in the Nāgarī script. MS Sc includes several interlinear vocalic corrections and in-line erasures. The *spaṣṭakrāntyādhikāra* appears on ff. 142v–144v, beginning in the middle of verse 1 at *yaṃ caika diśi*... on line 5 of f. 142v. The verses are numbered from 1 to 13 in a regular order, with the concluding verse of the chapter unnumbered on line 1 of f. 144v.

2.2 FORMAT OF THE EDITION

The orthographic standards and typographic conventions followed in preparing the edition of Nityānanda's *Sarvasiddhāntarāja* I.*spa·krā* are listed below. These conventions are described in Misra (2016: pp. 92–99) in greater detail.

1. In the edited text, folio breaks (corresponding to the beginning of a folio) are indicated by ⌈ with apposite folio numbers and manuscript sigla in the adjacent margin.
2. The *bhūtasaṃkhyā* numbers (word-numerals) and their corresponding digits are retained in the edition as they jointly appear in a few manuscripts.
3. Editorial additions to the text are indicated with words enclosed in angle brackets < >.
4. In the critical apparatus, the edited text (lemma) is separated from its corresponding variants by a right square-bracket].
5. The sigla of different manuscripts containing the same variant reading are separated by *commas*, whereas different variants (from different manuscripts) corresponding to the same lemma are separated by *semicolons*.
6. Fragments of Sanskrit words or compounds in Nāgarī are indicated with a small circle ∘ at their break-point. Extended string of words in the lemma are internally abbreviated with ∘...∘ between the letters.
7. Scribal corrections and erasures are shown with a diagonal line through the letter(s), e.g., ~~रि~~; vocalic corrections are shown with a horizontal line, e.g., णो indicating णो is corrected to णे by a scribe.
8. Interlinear insertion marks (*kākapada*) ˇ or ‸ seen in the manuscripts are identically reproduced in the variants of the critical apparatus.

9. Common scribal variations of Nāgarī orthography are emended silently without noting them in the critical apparatus, except where the grammatical meaning of the original reading may be ambiguous. These include: *anusvāra* for conjoined nasal consonants; omitted *visargas*, *virāmas*, and *avagrahas*; misplaced *daṇḍas*; ill-formed Nāgarī Sanskrit letters (e.g., य्र for य) and ill-formed vocalic marks (e.g., र्दी); irregular use of doubled consonant after a vowel-suppressed *r*-consonant (e.g., द्ध in अर्द्ध) or across line (*pāda*) breaks in a stanza; reversed conjunct consonants (e.g., ध्ब for ब्ध); and commonly confused consonant pairs (e.g., ब and व, प and य, त and न, ष and ख, etc.) and consonant graphemes (e.g., ष्ट for ष्ठ, etc.)
10. A few abbreviated Latin expressions are used to describe the variants in the critical apparatus. These include:
    - *corr.* for *correctio* 'correction',
    - *ins.* for *inseruit* 'inserted',
    - *om.* for *omisit* 'omitted',
    - *in ras.* for *in rasura* 'on top of an erasure',
    - *in marg. sins.* for *in margine sinsitro* 'in the left margin', and
    - *in marg. dext.* for *in margine dextro* 'in the right margin'.

2.3 FORMAT OF THE GLOSSARY

All Sanskrit words in the glossary are written with Nāgarī letters and are accompanied by corresponding Roman transliterations enclosed in parentheses. The technical expressions listed in the glossary are derived from § 3 where they appear as highlighted entries in the English translations.

- Equivalent Sanskrit terms are grouped together under their common technical translation in English, separated from each other by a semicolon. For example,

  maximum latitude **पर-इषु** (*para-iṣu*) 6, 10; **पर-शर** (*para-śara*) 10.
- At the end of each entry, I provide the appropriate verse-number to identify its location (in the English translation) of the text. For instance, in the example above, **पर-इषु** (*para-iṣu*) appears in verses 6 and 10 of § 3 (English translations on pp. 97 and 99), while **पर-शर** (*para-śara*) appears in verse 10 of § 3 (English translation on p. 99).
- References to multiple verse numbers are separated by commas. For example,

  sum **संयुति** (*saṃyuti*) 1, 10

  indicates that **संयुति** (*saṃyuti*) appears in verses 1 and 10 in § 3 (English translations on pp. 95 and 99).
- Mutually related technical translations in English are grouped together based on their linguistic or mathematical similarity. For instance, the head-

ing ecliptic poles— (on p. 166) includes the expressions ↪ ecliptic pole and ↪ pair of ecliptic poles.

- The glossary entries are arranged following the English alphabetical order.

3 *SARVASIDDHĀNTARĀJA*, I.*SPA·KRĀ*: TEXT AND TRANSLATION

[a]⌈ f.62r Bn.I
f.62v Bn.II
f.34r Br
f.71v Np
f.40r Pm
f.45r Rr

[b]⌈ f.142v Sc
[c]⌈ f.34v Br

[a]⌈खगस्य बाणो ऽन्यतमो ऽपमः पुनर्यदा द्व[b]⌈यं चैकदिशि[c]⌈स्थितं भवेत् ॥
तदा द्वयोः संयुतिरन्यथान्तरं स्फु⌈टापमाङ्काख्य इहोच्यते स्वदिक् ॥ १ ॥ ⌈ f.63r Bn.II

स्फुटापमाङ्कसिञ्जिनी सभत्रयद्युजीवया ॥
हता त्रिभज्यकोद्धृ⌈ता स्फुटापमज्यका भवेत् ॥ २ ॥ ⌈ f.143r Sc

परम⌈क्रान्तिकोटिज्या स्फुटक्रान्त्यङ्कजीवया ॥ ⌈ f.45v Rr
⌈हतान्यक्रान्तिकोटिज्याप्ता स्यात्स्पष्टापमज्यका ॥ ३ ॥ ⌈ f.40v Pm

अथ प्रकारान्तरेण ॥

कदम्बयुगलध्रुवद्वयमुपैति वृत्तं तु य⌈त्तदायनमुदीरितं ध्रुवचतुष्कयातं तथा ॥ ⌈ f.72r Np
नभोगविषुवद्वयोपरि पतत्सुवृत्तं च यद्भचक्रसदृशाह्वयं तदिति कल्पयेद्गोलवित् ॥ ४ ॥

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1 खगस्य०...०यदा द्व] *om.* Sc
1 दिशि०] दितिशि० Sc
2 ०माङ्कख्य] ०मा~~ख्यौ~~काख्य *corr.* Pm
2 इहोच्यते] ~~रि~~[इ]होच्यते *corr. in ras.* Pm
2 स्वदिक्] बुधैः Bn.I
2 १] ५८ Bn.I
3 सभत्रय०] सभव्त्रय० Bn.I
4 स्फुटापम०] यम० Bn.II; स्फुटोपम० *corr.* Pm
4 २] ५९ Bn.I
5 ०क्रान्त्यङ्क०] ०क्रांक० Bn.I
6 ०क्रान्तिकोटि०] ०कोटिक्रांति० Np, Sc
6 स्यात्स्प०] वास्प० Bn.II, Br, Pm, Rr
6 ३] ६० Bn.I
8 द्वयमुपैति०...०रितं ध्रुव] *om.* Sc
8 तदायन०] त्तदापयन० Pm
8 ०चतुष्क०] ०चतुक्क० Br
8 तथा ॥] तथा ४ Bn.II, Br, Pm, Rr; तया Sc
9 ०विषुवद्व०] ०विषु~~प्र~~वद्व० *corr.* Bn.II; ०विषुवम्द्व० Br; ०विषुवद्व० Np, Sc
9 पतत्सुवृत्तं] गतन्तुवृत्तं Np; यातस्तुवृत्तं Sc
9 च] तु Sc
9 ०सदृशा०] ०मदृशा० Br
9 तदिति] तदित Bn.II, Pm, Rr
9 कल्पयेद्गोलवित्] कल्पये[द्]गोलवित् *ins.* Sc
9 ४] ६१ Bn.I; ५ Bn.II, Br, Pm, Rr

[Given] the latitude of a celestial object (*khagasya bāṇa*), [and] again, the other declination (*anyatama-apama*) [i.e., the second declination]: if indeed both should be situated in one direction (*eka-diś*), then [we take] the sum (*saṃyuti*) of the two of them; otherwise, [we take their] difference (*antara*). [The result] is known as the curve of true declination (*sphuṭa-apama-aṅka*). Here, [it is] said to be [in] its own direction (*sva-diś*). 1

The Sine of the curve of true declination (*sphuṭa-apama-aṅka-siñjinī*), having been multiplied (*hatā*) by the day-Sine [of the longitude] increased by three zodiacal signs (*sa-bha-traya-dyujīvā*) [i.e., by the Cosine of the first declination of the 'longitude increased by 90°'] [and] having been divided (*uddhṛtā*) by the Radius (*tribhajyakā*) [i.e., by the *sinus totus*] should be the Sine of the true declination (*sphuṭa-apama-jyakā*). 2

The Cosine of the greatest declination (*parama-krānti-koṭijyā*) [i.e., the Cosine of the ecliptic obliquity], having been multiplied (*hatā*) by the Sine of the curve of true declination (*sphuṭa-krānti-aṅka-jīvā*) [and] having been divided (*āptā*) by the Cosine of the other declination (*anya-krānti-koṭijyā*) [i.e., by the Cosine of the second declination], should be the Sine of the true declination (*spaṣṭa-apama-jyakā*). 3

Now, with another method.

Now what circle (*vṛtta*) reaches the pair of ecliptic poles (*kadamba-yugala*) and the pair of celestial poles (*dhruva-dvaya*), that has been stated to be the solstitial [colure] (*āyana*[-*vṛtta*]) and also [as] the [circle] passing through the four poles (*dhruva-catuṣka-yāta*[-*vṛtta*]). And passing over a celestial object (*nabhoga*) and the pair of equinoctial points (*visuvat-dvaya*), what [circle] is well rounded (*su-vṛtta*), that the knower of spheres (*gola-vid*) should consider as the [circle] congruent to the ecliptic (*bhacakra-sadṛśa*[-*vṛtta*]) by name. 4

विषुववृत्तभवृत्तसदृक्षयोर्विवरगं धनुरायनवृत्तजम् ॥
भवति यत्कथितः स परस्फुटापम इति द्युचरस्य च तत्क्षणे ॥ ५ ॥

भवनचक्रभचक्रसदृक्षयोर्विवरगं धनुरायनवृत्तगम् ॥
भवति यत्स परेषुरिहोदितो विषुवपातयुगे सति क⌈ल्पिते ॥ ६ ॥ ⌈ f.143v Sc

विषुवन्नभोगमध्ये यत्कोदण्डं भवृत्तसदृशस्य ॥
ज्ञेयः सदृग्भुजाख्यो भायनविवरे सदृक्कोटिः ॥ ७ ॥

खगस्य कोटिसिञ्जिनी स्वबाणकोटिजीवया ॥
हता त्रिभज्यकोद्धृता सदृक्षकोटिसिञ्जिनी ॥ ८ ॥

तद्धनुर्नवति ९० तश्च्युतं यदा ⌈जायते सदृशबाहुसंज्ञकम् ॥ ⌈ f.62v Bn.I
या नभोगविशिखस्य सिञ्जिनी भाजिता ऽधरसदृक्षदोर्ज्यया ॥ ९ ॥

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1 ०रायन०] ०रायम० Br
2 यत्कथितः] यत्कथिः Bn.I
2 सपरस्फु०] परमस्फु० Sc
2 तत्क्षणे] तत्क्षणो corr. Pm
2 ५] ६२ Bn.I; ६ Bn.II, Br, Pm, Rr
3 ०भचक्रसदृक्षयोर्वि०] ०सदृक्षयोवि० Pm
3 ०वृत्तगम्] ०वृत्तयं Bn.I; ०वृत्तजं Bn.II, Br, Pm, Rr
4 यत्स परे०] त्सपरे० Br
4 ०युगे] ०यु।गे Bn.II
4 कल्पिते] कल्पिते ६ *ins.* Sc
4 ६] ६३ Bn.I; ७ Bn.II, Br, Pm, Rr
5 ०नभोगमध्ये] ०नभोंगमध्ये Bn.II, Pm, Rr
5 यत्कोदण्डं] यत्कों‸डं -दं- *ins. in marg. dext.* Br
5 भवृत्तस०] भवृत्तस्यस० Np; वृत्तस० Rr
6 सदृग्भुजाख्यो] सदृग्भुज्याख्यो Bn.I; सदृग्भुजाख्यौ Bn.II; सदाभुजाख्यो Np, Sc
6 भायन०] ग्रहादिबिंबंयन० Bn.II
6 सदृक्कोटिः] सदृत्कोटिः Bn.II, Pm, Rr
6 ७] ६४ Bn.I; ८ Bn.II, Br, Pm, Rr
7 स्वबाण०] स्वणबा० Bn.I
8 ०कोद्धृता] ०कोध्रता Bn.I
8 ०कोटि०] ०के-ा-टि *ins. in marg. dext.* Bn.II
8 ८] ६५ Bn.I; ९ Bn.II, Br, Pm, Rr
9 ९०] ओम्. Bn.I, Br, Pm, Rr, Sc
9 तश्च्युतं] तश्चुतं Pm
9 यदा] द्यदा Pm
9 जायते] यायते Bn.I
10 नभोग०] मभोग० Bn.II
10 ०विशिखस्य] ०विशिशिषस्य Bn.I; ०विशि ५ खस्य Bn.II, Pm, Rr
10 भाजिता ऽधरसदृक्षदोर्ज्यया] भाजिताधरसषष्टिभक्त-भाजकभजनमत्राधारभजनसंज्ञमच्यतेदोर्ज्याया Bn.II
10 ०दोर्ज्यया] ०दोर्ज्यका Bn.I
10 ९] ६६ Bn.I; १० Bn.II, Br, Pm, Rr

What arc (*dhanus*) produced on the solstitial colure (*āyana-vṛtta*) becomes situated in the difference (*vivara*) between the celestial equator (*viṣuva-vṛtta*) and the [circle] congruent to the ecliptic (*bhavṛtta-sadṛkṣa*[*-vṛtta*]), that is the stated [arc of] maximum true declination (*para-sphuṭa-apama*) of the celestial object (*dyucara*) just at that very moment. 5

What arc (*dhanus*) belonging to the solstitial colure (*āyana-vṛtta*) becomes situated in the difference (*vivara*) between the ecliptic (*bhavana-cakra*) and the [circle] congruent to the ecliptic (*bhacakra-sadṛkṣa*[*-vṛtta*]), in this case, that is the declared [arc of] maximum latitude (*para-iṣu*) when the conjunction of the equinoctial point and the node of the orbit [of the celestial object] (*viṣuva-pāta-yuga*) has been supposed. 6

What arc (*kodaṇḍa*) of the [circle] congruent to the ecliptic (*bhavṛtta-sadṛśa*[*-vṛtta*]) is between the equinoctial point (*viṣuvat*) and the celestial object (*nabhoga*), [that arc] should be known as the congruent arc (*sadṛś-bhujā*) by name; [and what is] between the celestial object (*bha*) and the solstitial colure (*āyana*[*-vṛtta*]), [that should be known as] the congruent complementary arc (*sadṛś-koṭi*) [i.e., the complement of *sadṛś-bhujā*]. 7

The Sine of the complement of the arc of longitude of a celestial object (*khagasya koṭi-siñjinī*), having been multiplied (*hatā*) by the Cosine of its latitude (*sva-bāṇa-koṭijīvā*) [and] having been divided (*uddhṛtā*) by the Radius (*tribhajyakā*) [i.e., by the *sinus totus*] (*tribhajyakā*), [should be] the Sine of the congruent complementary arc (*sadrkṣa-koṭi-siñjinī*) [i.e., the Sine of the complement of the *sadṛś-bhujā*]. 8

When [the measure of] its arc (*dhanus*), having been reduced from ninety [degrees] (*navatitaś-cyuta*), is determined, [it] has the name congruent arc (*sadṛśa-bāhu*). Or, what is the Sine of the latitude of a celestial object (*nabhoga-viśikhasya siñjinī*), having been divided (*bhājitā*) by the lowered Sine of the congruent arc (*adhara-sadrkṣa-dor-jyā*), 9

तद्धनुः परशराह्वयो भवेद्वा परेषुपरमापमाख्ययोः ॥
संयुतिर्वियुतिरस्ति च क्रमाद्गोलबाणसमभिन्नदिक्तया ॥ १० ॥

स ग्रहस्य परमस्फुटापमो जायते युतिवियोगदिक्स्थितः ॥
एवमभ्रनव ९०⌈तो ऽधिको यदा खाष्टभू १८० परिमितेर्विशोधितः ॥ ११ ॥ ⌈ f.63v Bn.II

परस्फुटक्रान्तिभवज्यका गुणा सद्दक्षबाहुज्यकया ऽधरीकृता ॥
तदीयचापं भवति स्फुटापमो दिगस्य संयोगवियोगदिक्समा ॥ १२ ॥

अन्यैर्यो बहुभिः प्रकारनिचयैः प्रोक्तः स्फुटापक्रमः
सत्स्थूलो यदुडु क्वचिद्भवलये तिष्ठत्कदंबाश्रितम् ॥
तद्बाणो नवतिः ९० सदैव परमक्रान्त्युत्थको द्यु⌈न्मिता ⌈ f.72v Np
स्पष्टक्रान्तिरिहास्ति गोलविदुषो गोलेऽपि संवीक्ष्यताम् ॥ १३ ॥

इत्येतस्यामिन्द्रपुर्यां वसन्सन्नित्यानन्दो देवदत्तस्य पुत्रः ।
सारोद्धारे सर्वसिद्धान्तराजे स्पष्टक्रान्तिं प्रापयत्तत्र ⌈पूर्तिम् ॥ ⌈ f.144v Sc

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1 परश०] परस० Bn.II, Rr
1 भवेद्वा] भवेद्वा Br
1 ०पमाख्ययोः] ०परमाख्ययोः Br
2 ०गोल०] ०होल० Pm
2 १०] ६७ Bn.I; ११ Bn.II, Br, Pm, Rr
4 ०नवतो ९०] ०नखतो २०० Np; ०नखतो Sc
4 यदा] य Bn.II
4 ०मितेर्विशो०] ०मितेविशो० Pm
4 ११] ६८ Bn.I; १२ Bn.II, Br, Pm, Rr
5 ०स्फुट०] ०स्फुद० Br
5 ०कया ऽधरीकृता] ०कयाषष्टिभजनमध्यरीकरणमुच्यतेता Bn.II; ०कथाधरीकृताः Pm, Rr
6 भवति] भव Pm
6 १२] ६९ Bn.I; १३ Bn.II, Br, Pm, Rr
7–10 अन्यैर्ये०…०संवीक्ष्यताम् ॥ १३ ॥] *om.* Bn.II, Br, Pm, Rr
7 अन्यैर्यो] अन्यैर्ये Np, Sc
7 प्रोक्तः स्फु०] प्रोक्तस्फु० Np; प्रोक्ताः स्फु० *corr.* Sc
8 सत्स्थूलो] सस्थूलो Bn.I, Sc
9 ०क्रान्त्युत्थको] ०क्रान्तुस्यको Bn.I
9 द्युन्मिता] द्युम्मिता Np
10 संवीक्ष्यताम्] सावीक्ष्यतां Bn.I
10 १३] ७० Bn.I
11 इत्येतस्या०] इत्येंतस्या० Bn.II
11 ०सन्नित्यानन्दो] ०सन्निनंदो -त्या- *ins. in marg. dext.* Pm; ०सननित्याद्रो -नन्- *ins. in marg. sins.* Sc
12 प्रापयत्तत्र] प्रापयतत्र Bn.II
12 पूर्तिम् ॥] पूर्त्तिं ॥ ७१ ॥ Bn.I; पूर्ति १४ Bn.II, Pm; पूर्तिं १४ Br, Rr; पूर्त्तिम् ॥ ९ ॥ Np

its arc (*dhanus*) should be [called] the maximum latitude (*para-śara*) by name. There is the sum (*saṃyuti*) or the difference (*viyuti*) of the two [quantities] known as the maximum latitude (*para-iṣu*) and the greatest declination (*parama-apama*) [i.e., the obliquity of the ecliptic] with the latitude (*bāṇa*) and the celestial hemisphere (*gola*) [i.e., the declination of the celestial object] in the same or different directions (*sama-bhinna-diś*) respectively. 10

That [result], being situated in the direction of the conjunction or the disjunction (*yuti-viyoga-diś*), becomes the maximum true declination of a celestial object (*grahasya parama-sphuṭa-apama*). Thus, when [its measure is] greater (*adhika*) than ninety [degrees] (*abhra-nava*), [it is] made to be subtracted (*viśodhita*) from a measure of one hundred and eighty [degrees] (*kha-aṣṭa-bhū*). 11

The Sine of the maximum true declination (*para-sphuṭa-krānti-bhava-jyakā*) multiplied (*guṇā*) by the Sine of the congruent arc (*sadṛkṣa-bāhu-jyakā*) [and] having been lowered (*adharī-kṛtā*), its arc (*cāpa*) becomes the true declination (*sphuṭa-apama*). Its direction (*diś*) is the same (*sama*) as the direction of the conjunction or the disjunction (*saṃyoga-viyoga-diś*). 12

What is declared by many others, in multitudes of ways, [as the] true declination (*sphuṭa-apakrama*), [that] is actually imprecise (*sthūla*). At some point, whichever star in the circle of asterisms (*bha-valaya*) remains stationed at the ecliptic pole (*kadamba*), the latitude (*bāṇa*) of that [star] is always ninety (*navati*) [degree]. What is derived from the greatest declination (*parama-krānti*) [i.e., the obliquity of the ecliptic], that, in this case, is the true declination (*spaṣṭa-krānti*) [of the star] measuring the day-[Sine] (*dyu*-[*jīvā*]) [i.e., the Cosine of the first declination of the longitude]. [This] should be seen in the very [exposition of the] sphere (*gola*) by wise men who know [the science] of spheres (*gola-viduṣa*). 13

In this manner, the wise Nityānanda, son of Devadatta, living in this city of Indrapurī, brought the [section on] true declination to completion in the quintessential *Sarvasiddhāntaraja* 'King of all *siddhānta*s'.

4 *SARVASIDDHĀNTARĀJA*, I.*SPA·KRĀ*: TECHNICAL ANALYSIS

The *Sarvasiddhāntarāja* I.*spa·krā* describes three methods to compute the true declination of a celestial object. Nityānanda explains these methods in eleven metrical verses that are taken from his *Siddhāntasindhu* Part II.6 (see Table 1). In the following subsections, I discuss the mathematics behind each of these methods, and include reference to other Islamicate and Sanskrit works where similar (or near-similar) methods have been attested. I begin my discussion by first describing the different geometrical objects and trigonometrical relations identified on the celestial sphere.

4.1 GEOMETRY ON THE CELESTIAL SPHERE

Figure 1 depicts the celestial sphere with a celestial object positioned at S above the celestial equator $\bigcirc T\Upsilon R\Omega$ and directed towards the (northern) celestial pole P. The ecliptic $\bigcirc T'\Upsilon R'\Omega$, with its pair of ecliptic poles P′ and $\overline{P'}$, is inclined to the celestial equator at the equinoctial points ♈ (i.e., 0° Aries) and Ω (i.e., 180° Libra) with obliquity ϵ.[23] The circle $\bigcirc P'PR'R$ is the solstitial colure.[24] The circle $\bigcirc N\Upsilon H\Omega$ is the great circle of a celestial sphere that passes through S and the vernal equinoctial point ♈ (and the autumnal equinoctial point Ω) and intersects the solstitial colure at points N and H. This circle is called the great circle congruent to the ecliptic.[25]

The position of the celestial object (at S) can be described with its ecliptic and equatorial coordinates as

$$\begin{array}{ll} \textit{Ecliptic coordinates} & \textit{Equatorial coordinates} \\ \widehat{SD} = \beta \text{ (latitude)} & \widehat{SA} = \delta \text{ (declination)} \\ \widehat{\Upsilon D} = \lambda \text{ (longitude)} & \widehat{\Upsilon A} = \alpha \text{ (right ascension)} \end{array} \tag{1}$$

23 On various occasions in his *Siddhāntasindhu* Part II.6 and *Sarvasiddhāntarāja* I.*spa·krā*, Nityānanda refers to the obliquity of the ecliptic as the greatest declination (*parama-krānti*). Mullā Farīd uses the Persian expression *mayl-i kullī* ‘total declination’ to refer to the ecliptic obliquity in his *Zīj-i Shāh Jahānī* Discourse II.6.

24 Nityānanda identifies the solstitial colure (*āyana-vṛtta*) in his *Siddhāntasindhu* Part II.6, $[\alpha]_{\text{verse}}$ and in his *Sarvasiddhāntarāja* I.*spa·krā*, verse 4, as the circle passing through the four poles (*dhruva-catuṣka-yāta-vṛtta*); in other words, the great circle passing through the two pairs of celestial and ecliptic poles. In the *Zīj-i Shāh Jahānī* Discourse II.6, passages [8], [9], and [11], Mullā Farīd also identifies the solstitial colure as *dāyiri-yi mārri bi aqṭāb-i arbaʿi* ‘circle passing through the four poles’ (see Misra 2021: pp. 88, 90, and 94).

25 Nityānanda calls this great circle the circle congruent to the ecliptic (*bhacakra-sadṛśa-vṛtta*) in his *Sarvasiddhāntarāja* I.*spa·krā*, verses 4–7, and also in his *Siddhāntasindhu* Part II.6, $[\alpha]_{\text{verse}}$–$[\delta]_{\text{verse}}$, as it resembles (*sadṛśa*) the ecliptic (*bhacakra*) in meeting the celestial equator at the two equinoctial points, and is inclined to the celestial equator (like the ecliptic) with changing inclination (see Misra 2021: pp. 94 and 96).

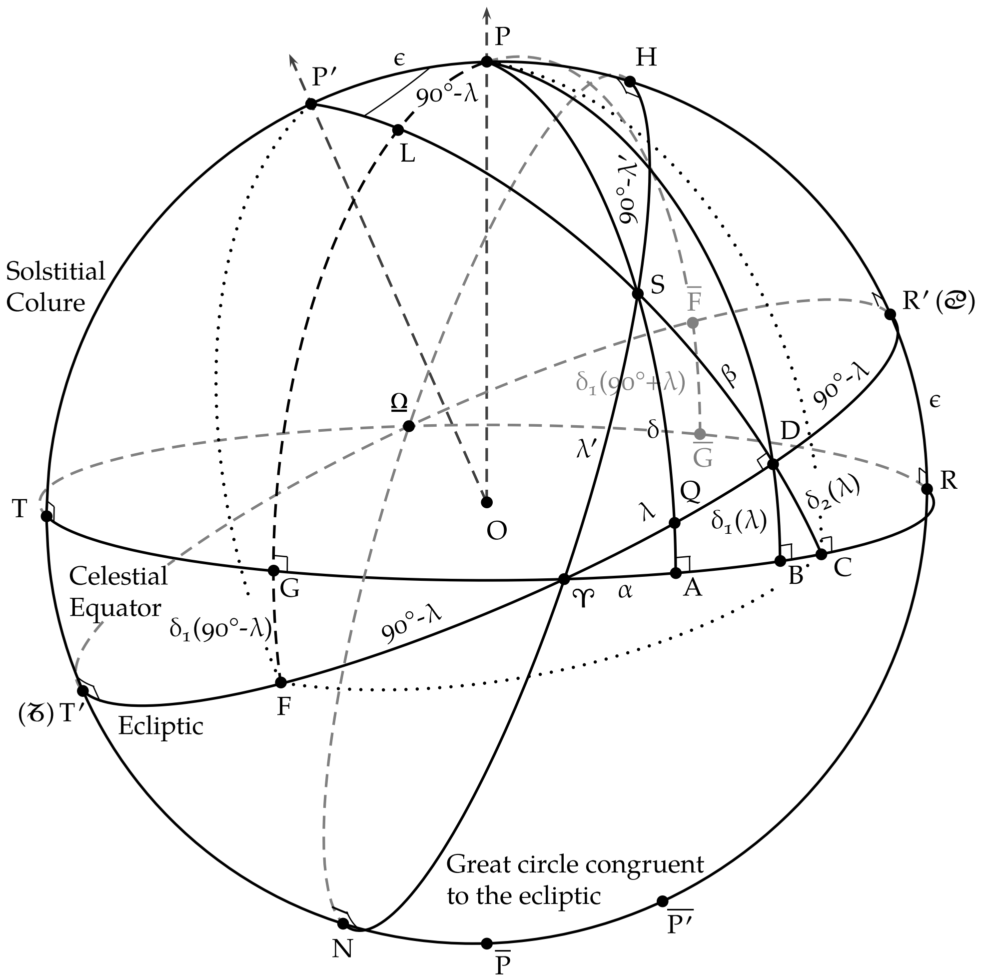


Figure 1: The celestial sphere showing the different spherical triangles inscribed by the celestial equator, the ecliptic, a great circle congruent to the ecliptic (and passing through the celestial object), and their different secondary circles.

where the ecliptical projection point D represents the point of intersection of the secondary to the ecliptic passing through the celestial object at S. Hence, $\sphericalangle SDT'$ or $\sphericalangle SDR' = 90^\circ$. Similarly, the equatorial projection point A represents the point of intersection of the secondary to the equator passing through the celestial object at S, and accordingly, $\sphericalangle SAT$ or $\sphericalangle SAR = 90^\circ$. The secondary to the ecliptic passing through the celestial object at S goes past its ecliptical projection point D and intersect the equator at C, with the arc $\widehat{P'SD} = 90^\circ$. Also, the secondary to the equator passes through the ecliptical projection point D and meets the celestial equator at point B, with the arc $\widehat{\mathrm{PDB}} = 90°$. These, then, allows us to define the arcs

$$\widehat{\mathrm{DB}} = \delta_1(\lambda) \text{ as the 'first declination' (simply called } \textit{krānti} \text{ or } \textit{apama}\text{) and} \tag{2}$$

$$\widehat{\mathrm{DC}} = \delta_2(\lambda) \text{ as the 'second declination' (}\textit{dvitīyā-krānti} \text{ or } \textit{anya-apama}\text{)} \tag{3}$$

corresponding to the arc of longitude $\widehat{\Upsilon\mathrm{D}} = \lambda$ of the celestial object at S. Appendix B discusses Nityānanda's rule for calculating the second declination from the first (in the *Sarvasiddhāntarāja* I.4.49–50ab).

Mullā Farīd, in his *Zīj-i Shāh Jahānī* Discourse II.6, passages [1] and [7], refers to the first and second declinations of the longitude as *mayl-i daraji-yi u* 'declination of its degree' and *mayl-i thānī-yi daraji-yi u* 'second declination of its degree' respectively, with the longitude of the celestial object simply called *daraji-yi u* 'its degree' (see Misra 2021: pp. 86 and 88).[26]

We can also consider two antipodal points F and $\overline{\mathrm{F}}$ on the ecliptic such that

$$\begin{gathered}\widehat{\mathrm{F}\Upsilon\mathrm{D}} = 90° \text{ and } \widehat{\mathrm{DR}'\overline{\mathrm{F}}} = 90°, \text{ with } \widehat{\Upsilon\mathrm{D}} = \lambda. \\ \text{Thus, } \widehat{\mathrm{F}\Upsilon} = 90° - \lambda \text{ and } \widehat{\overline{\mathrm{F}}\Upsilon} = 90° + \lambda.\end{gathered} \tag{4}$$

This helps us define the arcs $\widehat{\mathrm{GF}}$ and $\widehat{\overline{\mathrm{GF}}}$ on the secondary to the equator (shown as the dashed arc $\widehat{\mathrm{GP}\overline{\mathrm{G}}}$ in Figure 1) that intersects points F and $\overline{\mathrm{F}}$ of the ecliptic respectively.

- The arc $\widehat{\mathrm{GF}}$ represents the first declination of the complement of the longitude of the celestial object; in other words,

$$\widehat{\mathrm{GF}} = \delta_1(90° - \lambda) \text{ or } \delta_1(\overline{\lambda}) \text{ where } \overline{\lambda} = 90° - \lambda. \tag{5}$$

- And arc $\widehat{\overline{\mathrm{GF}}}$ represents the first declination of the longitude of the celestial object increased by 90°; in other words,

$$\widehat{\overline{\mathrm{GF}}} = \delta_1(90° + \lambda). \tag{6}$$

In the *Siddhāntasindhu* Part II.6, $[2]_{\text{verse}}$ and the *Sarvasiddhāntarāja* I.*spa·krā*, verse 2, Nityānanda refers to the argument (90° + λ) as *sa-bha-traya* '[longitude] increased by three signs', see § 4.2.2. Mullā Farīd, in the *Zīj-i Shāh*

26 The partial first and second declinations are called *al-māyl al-awwal al-juzʾī* and *al-māyl al-thānī al-juzʾī* respectively in Arabic, see Kūshyār b. Labbān's *al-Zīj al-Jāmiᶜ* (c. 1020–1025), chapter III (book 31): *al-bāb al-mufrad fī jawāmiᶜ ᶜilm al-hayᶜa* 'A special chapter on generalities of the science of cosmology', Bagheri (2006).

Jahānī Discourse II.6, passage [2], calls $\delta_1(90° + \lambda)$ as the *mayl-i mankūs-i daraji-yi kawkab* 'inverse declination of the degree of the celestial object' (see Misra 2021: pp. 86 and 92).

- The equal inclinations of the northern and southern halves of ecliptic (with respect to the celestial equator) suggest that the first declinations of antipodal ecliptic points F and $\overline{\mathrm{F}}$ are equal. In other words,

$$\widehat{\mathrm{GF}} = \widehat{\mathrm{G\overline{F}}} \text{ or } \delta_1(90° - \lambda) = \delta_1(90° + \lambda). \tag{7}$$

4.1.1 *List of triquadrantal spherical triangles*

Looking at the various secondaries to the ecliptic and the equator in Figure 1, we can identify the following triquadrantal spherical triangles on the celestial sphere:[27]

$\triangle$ P′DF where $\widehat{\mathrm{P'D}}$, $\widehat{\mathrm{DF}}$, and $\widehat{\mathrm{P'F}}$ are all 90°;

$\triangle$ LCF where $\widehat{\mathrm{LC}}$, $\widehat{\mathrm{CF}}$, and $\widehat{\mathrm{LF}}$ are all 90°; and

$\triangle$ PCG where $\widehat{\mathrm{PC}}$, $\widehat{\mathrm{CG}}$, and $\widehat{\mathrm{PG}}$ are all 90°.

4.1.2 *List of complementary arcs*

Also, the construction of various secondaries to the ecliptic and the equator in Figure 1 allows us to identify the following complementary arcs on the celestial sphere:

- the arc of the complement of ecliptic obliquity $\widehat{\mathrm{PR'}} = \widehat{\mathrm{PR}} - \widehat{\mathrm{R'R}} = 90° - \epsilon$ where the arc of ecliptic obliquity $\widehat{\mathrm{R'R}}$ $(= \widehat{\mathrm{P'P}}) = \epsilon$,
- the arc of the co-latitude $\widehat{\mathrm{P'S}} = \widehat{\mathrm{P'D}} - \widehat{\mathrm{SD}} = 90° - \beta$ where the arc of the latitude $\widehat{\mathrm{SD}} = \beta$,
- the arc of co-longitude $\widehat{\mathrm{R'D}} = \widehat{\Upsilon\mathrm{R'}} - \widehat{\Upsilon\mathrm{D}} = 90° - \lambda$ where the arc of the longitude $\widehat{\Upsilon\mathrm{D}} = \lambda$,
- the arc of the co-declination $\widehat{\mathrm{PS}} = \widehat{\mathrm{PA}} - \widehat{\mathrm{SA}} = 90° - \delta$ where the arc of the declination $\widehat{\mathrm{SA}} = \delta$,

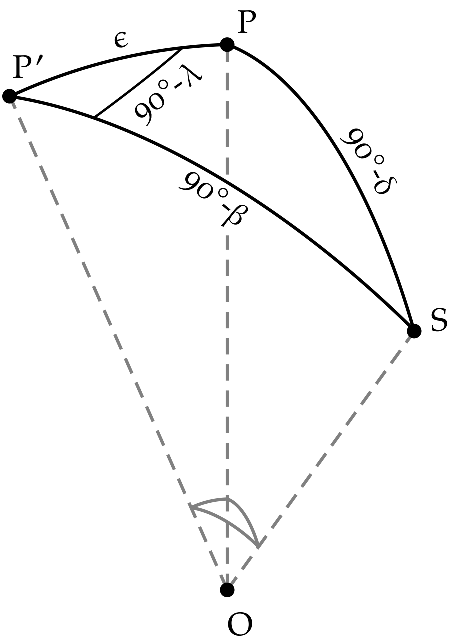
27 Point P′ is the pole with respect to the points D and F (on the ecliptic) that are mutually separated by 90°, and hence all sides of the spherical triangle $\triangle$P′DF measure ninety degrees. Moreover, point F is the pole with respect to the points L and C (on the secondary to the ecliptic) that are also mutually separated by 90° (since $\widehat{\mathrm{P'L}} = \widehat{\mathrm{DC}} = \delta_2(\lambda)$), and hence all sides of the spherical triangle $\triangle$LCF also measure ninety degrees. And lastly, with point C being the pole of the points P and F that lie on a secondary to the celestial equator passing through G, it is evident that all sides of the spherical triangle $\triangle$PCG also measure ninety degrees.

- the arc of the co-'first' declination $\overset{\frown}{PD} = \overset{\frown}{PB} - \overset{\frown}{DB} = 90° - \delta_1(\lambda)$ where the arc of the first declination $\overset{\frown}{DB} = \delta_1(\lambda)$,
- the arc of the co-'second' declination $\overset{\frown}{LD} = \overset{\frown}{LC} - \overset{\frown}{DC} = 90° - \delta_2(\lambda)$ where the arc of the second declination $\overset{\frown}{DC} = \delta_2(\lambda)$, and
- the arc of the complement of the 'first declination of the complement of the longitude' $\overset{\frown}{LG} = \overset{\frown}{LF} - \overset{\frown}{GF} = 90° - \delta_1(\overline{\lambda}) = \overline{\delta_1(\overline{\lambda})}$ where the arc of the first declination of the complement of the longitude $\overset{\frown}{GF} = \delta_1(90° - \lambda) = \delta_1(\overline{\lambda})$.

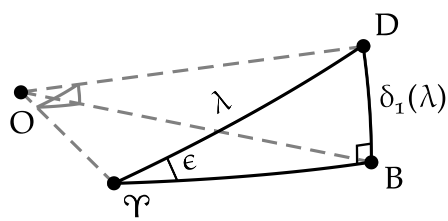
4.1.3 *Computing the true declination of a celestial object, the modern method*

The true declination δ of a celestial object at S (in Figure 1) can be computed by applying the spherical law of cosines to the spherical triangle $\triangle P'PS$ formed by the ecliptic pole P′, the celestial pole P, and the celestial object at S as follows.

In the excerpt (from Figure 1) shown below to the left, $\overset{\frown}{P'P} = \epsilon$, $\overset{\frown}{P'S} = 90° - \beta$, $\overset{\frown}{PS} = 90° - \delta$, and $\sphericalangle PP'S = \overset{\frown}{R'D} = 90° - \lambda$. By applying the spherical law of cosines,[28] we find



Excerpt of $\triangle P'PS$ from Figure 1.



Excerpt of $\triangle DB♈$ from Figure 1.

28 For discussions on the history of trigonometry see Debarnot (1996) and Van Brummelen (2013). Also see Sengupta (1931) for a comparison between Greek and Indian methods of spherical trigonometry in the context of astronomy.

$\cos \widehat{PS} = \cos \widehat{PP'} \cdot \cos \widehat{P'S} + \sin \widehat{PP'} \cdot \sin \widehat{P'S} \cdot \cos \sphericalangle PP'S$, in other words, $\cos(90° - \delta) = \cos\epsilon \cdot \cos(90° - \beta) + \sin\epsilon \cdot \sin(90° - \beta) \cdot \cos(90° - \lambda)$ or

$$\sin\delta = \cos\epsilon \cdot \sin\beta + \sin\epsilon \cdot \cos\beta \cdot \sin\lambda. \tag{8}$$

Now, as the excerpt (from Figure 1) shown above to the right indicates, the right spherical triangle $\triangle DB\Upsilon$ has $\widehat{DB} = \delta_1(\lambda)$, $\widehat{\Upsilon D} = \lambda$, and $\sphericalangle D\Upsilon B = \widehat{R'R} = \epsilon$. And hence, applying the spherical law of sines to triangle $\triangle DB\Upsilon$, we have,[29]

$$\frac{\sin \widehat{DB}}{\sphericalangle D\Upsilon B} = \frac{\widehat{\Upsilon D}}{\sphericalangle \Upsilon BD} \Rightarrow \frac{\sin\delta_1(\lambda)}{\sin\epsilon} = \frac{\sin\lambda}{\sin 90°} \text{ or,}$$
$$\sin\delta_1(\lambda) = \sin\epsilon \cdot \sin\lambda. \tag{9}$$

Thus, from equations (8) and (9), we have

$$\sin\delta = \cos\epsilon \cdot \sin\beta + \cos\beta \cdot \sin\delta_1(\lambda) \tag{10}$$

from which the true declination δ can be readily computed, knowing the latitude β and the first declination $\delta_1(\lambda)$ of the celestial object (at S), and the ecliptic obliquity ϵ.

4.2 PRELIMINARY DEFINITIONS OF CONSTITUENT ARCS

In his statements on the three methods to compute the true declination of a celestial object, Nityānanda refers to certain arcs of great circles on the celestial sphere, as well as their corresponding measures, by translating their Arabic or Persian names into Sanskrit. Some of these terms are not commonly known in Sanskrit astronomy, and hence, I include a preliminary description of these quantities in the following subsections.

4.2.1 Curve of true declination

The curve of true declination (*sphuṭa-apama-aṅka*) is the distance of a celestial object from the celestial equator measured along a secondary circle to the ecliptic that passes through the body of the celestial object. In the chapter on spheres (*golādhyāya*) in his *Sarvasiddhāntarāja* (II.117), Nityānanda defines the curve of true declination expressly:

29 The method of computing the (sine of the) declination of a point on the ecliptic as a product of the (the sines) of ecliptic obliquity and the longitude of that point can be found in Ptolemy's *Almagest* (c. second century CE), Book I.14 (see Toomer 1984: 69–70). Ptolemy's expression for the 'method of declination', however, is expressed in terms of chords (instead of sines) and relies on Menelaus' proposition III.1 (first form) from his *Sphaerica* (see Neugebauer 1975: Theorem I on p. 28).

विषुवमण्डलखेचरमध्यगं विशिखसूत्रधनुर्यदिहास्ति सः ॥
स्फुटतरापमकाङ्क उदीरितो गणितगोलविचारविचक्षणैः ॥ ११७ ॥

viṣuvamaṇḍalakhecaramadhyagaṃ viśikhasūtradhanur yad ihāsti saḥ ||
sphuṭatarāpamakāṅka udīrito gaṇitagolavicāravicakṣaṇaiḥ || 117 ||

What arc (*dhanus*) of the direction [circle] of latitude (*viśikha-sūtra*) is here between the celestial equator (*viṣuva-maṇḍala*) and the celestial object (*khecara*), that [arc] is declared as the curve of true declination (*sphuṭatara-apama-aṅka*) by those who are wise (*vicakṣaṇa*) in the investigations (*vicāra*) of computations (*gaṇita*) and spheres (*gola*). 117

In verse 1 of the *Sarvasiddhāntarāja* I.*spa·krā*, he adds to this definition, saying

खगस्य बाणो ऽन्यतमो ऽपमः पुनर्यदा द्वयं चैकदिशिस्थितं भवेत् ॥
तदा द्वयोः संयुतिरन्यथान्तरं स्फुटापमाङ्काख्य इहोच्यते स्वदिक् ॥ १ ॥

khagasya bāṇo 'nyatamo 'pamaḥ punar
yadā dvayaṃ ca ikadiśisthitaṃ bhavet ||
tadā dvayoḥ saṃyutir anyathāntaraṃ
sphuṭāpamāṅkākhya ihocyate svadik || 1 ||

[Given] the latitude of a celestial object (*khagasya bāṇa*), [and] again, the other declination (*anyatama-apama*) [i.e., the second declination]: if indeed both should be situated in one direction (*eka-diś*), then [we take] the sum (*saṃyuti*) of the two of them; otherwise, [we take their] difference (*antara*). [The result] is known as the curve of true declination (*sphuṭa-apama-aṅka*). Here, [it is] said to be [in] its own direction (*sva-diś*). 1

Nityānanda's statements allow us to conceive the curve of true declination as the arc $\widehat{SC}$ in Figure 1. The arc $\widehat{SC}$ is equal to the sum or difference of the arcs of the second declination $\widehat{DC}$ and the latitude $\widehat{SD}$ depending on their mutual orientations. Figure 2 depicts the two configurations of the celestial object (at S) in relation the celestial equator and the ecliptic such that

1. when the arcs of second declination $\widehat{DC}$ and the latitude $\widehat{SD}$ as similarly oriented, i.e., both lying to the north (or both lying to the south) of the celestial equator and the ecliptic respectively, the curve of true declination $\widehat{SC}$ is $\widehat{DC} + \widehat{SD}$ or $\delta_2(\lambda) + \beta$, see Figure 2a; and
2. when the arcs of second declination $\widehat{DC}$ and the latitude $\widehat{SD}$ as differently oriented, i.e., both lying in opposing hemispheres with respect to the celestial equator and the ecliptic respectively, the curve of true declination $\widehat{SC}$ is $\widehat{DC} - \widehat{SD}$ or $\delta_2(\lambda) - \beta$, see Figure 2b.

Thus, more generally

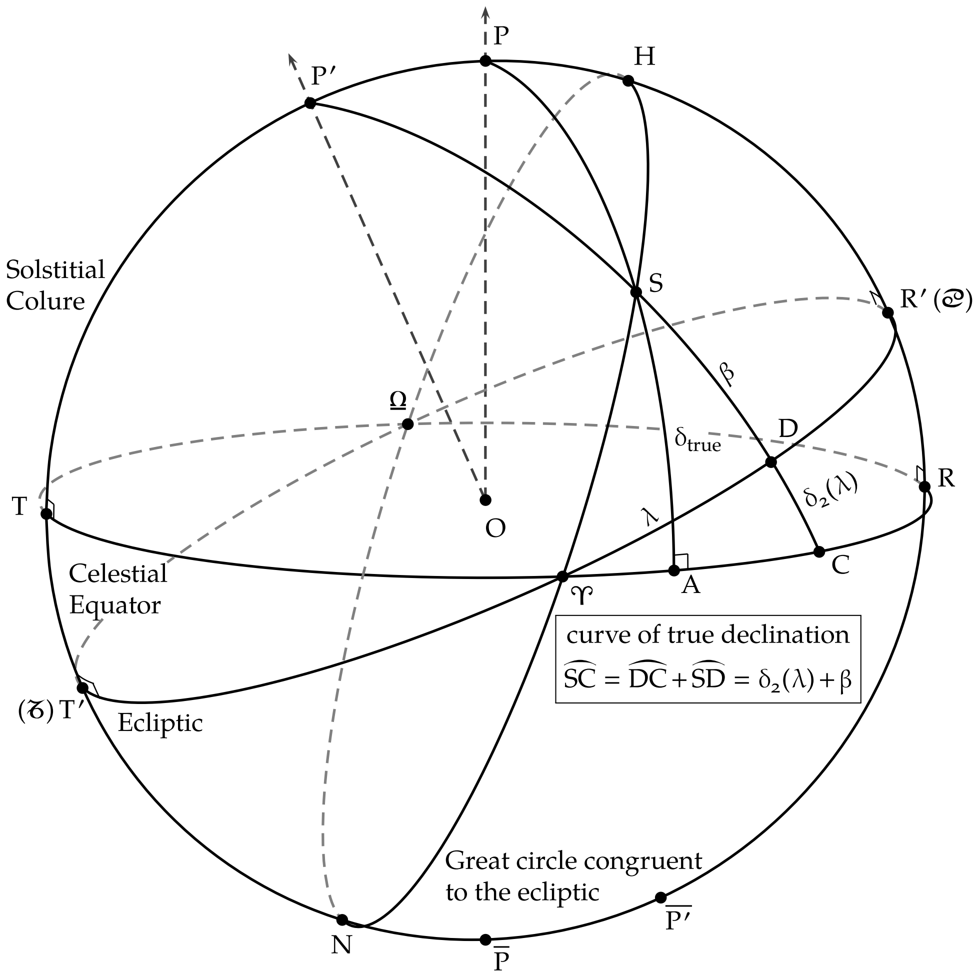
$$\textit{sphuṭa-apama-aṅka} \text{ or } \widehat{SC} = \widehat{DC} \pm \widehat{SD} = \delta_2(\lambda) \pm \beta. \tag{11}$$

There are two grammatical points of note in verse 1:

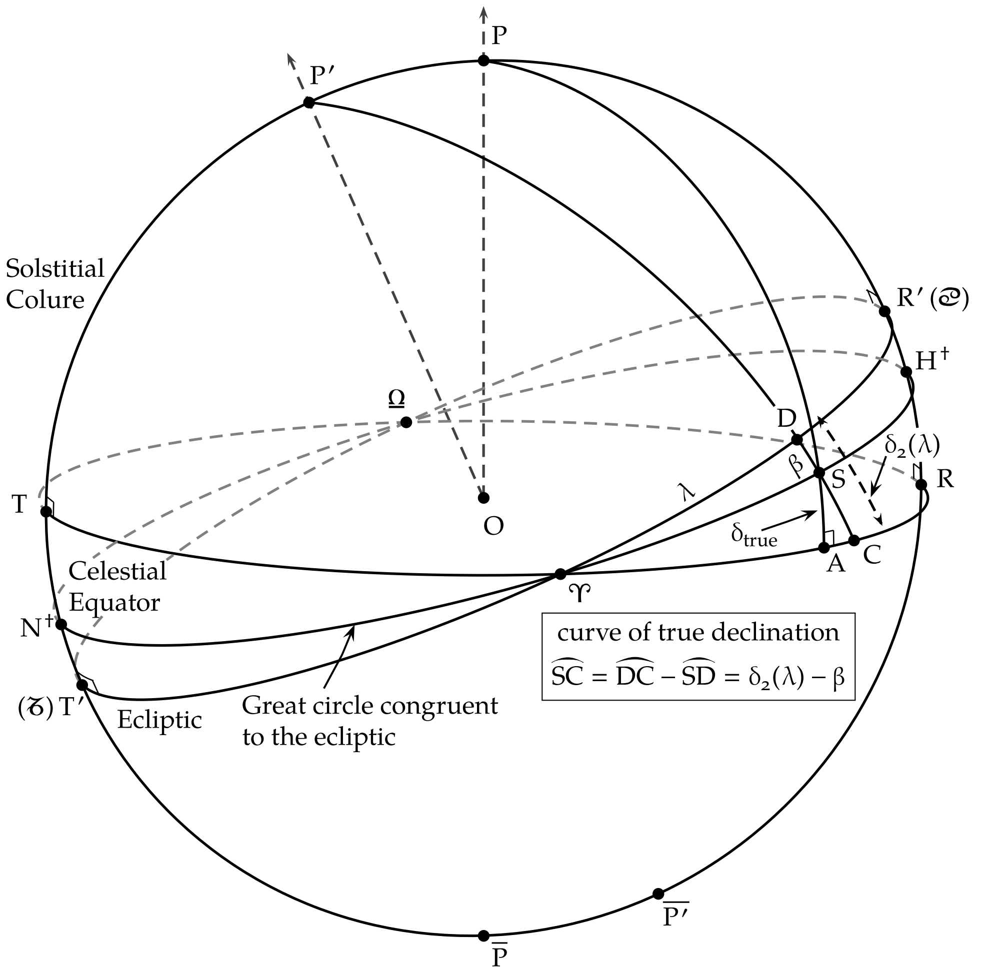
1. In the *Siddhāntasindhu* Part II.6, $[1]_{\text{verse}}$, Nityānanda translates the Persian expression *ḥiṣṣi-yi buᶜd* 'share of the distance' from Mullā Farīd's *Zīj-i Shāh Jahānī* Discourse II.6, passage [1] as *sphuṭa-apama-aṃśa* 'share of the true declination' (see Misra 2021: pp. 86 and 92).[30] In his *Sarvasiddhānta-rāja* I.*spa·krā*, verse 1, he changes this near-literal Sanskrit translation of a Persian expression to *sphuṭa-apama-aṅka* 'curve of true declination'. Geometrically, all three expressions refer to the arc $\widehat{SC}$ of equation (11).
2. Ordinarily, the Sanskrit word *aṅka* implies a number, measure, or mark; however, in this specific technical context, I take this word to mean the curved side of a figure, i.e., an arc of a great circle.[31] In various places in *Siddhāntasindhu*, Nityānanda uses the word *aṅka* in the expression for the 'curve of true declination' (*sphuṭa-apama-aṅka*). The word *aṅka* also appears in the *golādhyāya* of his *Sarvasiddhāntarāja* (II.117, stated above) where he defines this geometrical quantity.
   I suspect the use of *aṅka* in the expression *sphuṭa-apama-aṅka* for the 'curve of true declination' (i.e., $\widehat{SC}$) may serve to differentiate it from ordinary expressions like *sphuṭa-apama-cāpa* that may be confused for the arc of true declination (i.e., $\widehat{SA}$). The use of the expression *sphuṭatara-apamaka-aṅka*, lit. curve of the 'truer' declination, in his *golādhyāya* further qualify the unique name of this geometrical quantity.

30 al-Kāshī refers to the quantity $\delta_2(\lambda) \pm \beta$, i.e., $\widehat{SC}$, as *ḥiṣṣa-yi ᶜarḍ* 'share of the latitude' (see Kennedy 1985: 9); whereas, Ulugh Beg calls it the *ḥiṣṣat al-buᶜd* (in Arabic) 'share of the distance' (see Sédillot 1853: 89).

31 In literary works, the word *aṅka* is sometimes used to indicate a curved line, e.g., the words *tri-rekhā-aṅka-kaṇtha* 'the three curved lines on the neck' in the *Dāmodarāṣṭaka*, verse 2 (from the *Padma Purāṇa*). Its near-similar equivalent *aṅkas* (comparable to the Greek ἄγκος and Latin *ancus*) directly implies a curve or bend, see, e.g., *Ṛgveda* (IV.40.4).



(a) *Similarly Oriented*



(b) *Differently Oriented*

Figure 2: The curve of true declination of a celestial object (at S) in two different configurations:

(a) *Similarly Oriented* The arc $\widehat{\mathrm{N}♈\mathrm{H}}$ represents a great circle congruent to the ecliptic and passing through the celestial object at a given instant for which the second declination $\delta_2(\lambda)$ and latitude β are similarly orientated in their respective hemispheres, i.e., $\delta_2(\lambda)$ and beta are both in the northern (or southern) hemispheres in relation to the celestial equator and the ecliptic respectively.

(b) *Differently Oriented* The arc $\widehat{\mathrm{N}^{\dagger}♈\mathrm{H}^{\dagger}}$ represents a great circle congruent to the ecliptic and passing through the celestial object at a given instant for which the second declination $\delta_2(\lambda)$ and the latitude β are oppositely oriented in their respective hemispheres, i.e., $\delta_2(\lambda)$ and β are both in opposing hemispheres in relation to the celestial equator and the ecliptic respectively.

4.2.2 *Day-Sine of the longitude*

The day-Sine of the longitude is the measure of the circle of day-Radius of a celestial object lying on the ecliptic; in other words, the Radius of the day-circle of the ecliptical projection of the object in the celestial sphere.

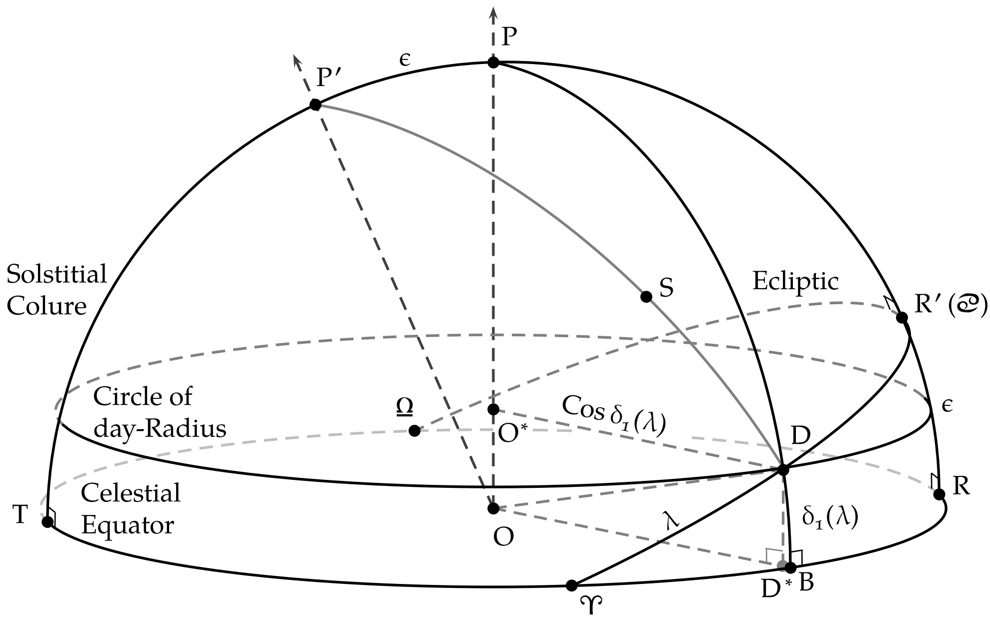


Figure 3: The circle of day-Radius corresponding to the ecliptical projection D of a celestial object (at S) with longitude λ and first declination $\delta_1(\lambda)$.

Figure 3 depicts a celestial object at S, at a particular time of the day, with longitude λ and first declination $\delta_1(\lambda)$. The day-Sine is then the measure of the Radius of the small circle passing through point D (ecliptical projection of the celestial object). Thus, with $\widehat{DB} = \sphericalangle DOD^* = \delta_1(\lambda)$ in right-angled $\triangle DOD^*$, we have

$$\widehat{DD^*} = \widehat{OD} \cdot \sin\delta_1(\lambda) = \mathrm{Sin}\,\delta_1(\lambda) \quad \text{and} \quad \widehat{OD^*} = \widehat{OD} \cdot \cos\delta_1(\lambda) = \mathrm{Cos}\,\delta_1(\lambda).$$

where $\widehat{OD} = \mathcal{R}$.[32] With $\widehat{OD^*} = \widehat{O^*D}$, the Radius of the day-circle (i.e., day-Radius), or equivalently,

$$\text{the day-Sine of the longitude } (dyujīvā) = \widehat{O^*D} = \mathrm{Cos}\,\delta_1(\lambda). \tag{12}$$

32 I capitalise trigonometric functions to indicate a non-unitary radius, i.e., Sin = $\mathcal{R}$sin and Cos = $\mathcal{R}$cos where the radius $\mathcal{R}$ is the Radius or *sinus totus* (i.e., sine of 90°). Mullā Farīd's *Zīj-i Shāh Jahānī* and Nityānanda's *Siddhāntasindhu* and *Sarvasiddhāntarāja* take $\mathcal{R} = 60$.

Correspondingly, the day-Sine [of the longitude] increased by three zodiacal signs is then equal to the Cosine of the first declination of the 'longitude increased by 90°', i.e.,

$$\text{the day-Sine [of the longitude] increased by three zodiacal signs } (\textit{sa-bha-traya-dyujīvā}) = \operatorname{Cos}\delta_1(90° + \lambda). \tag{13}$$

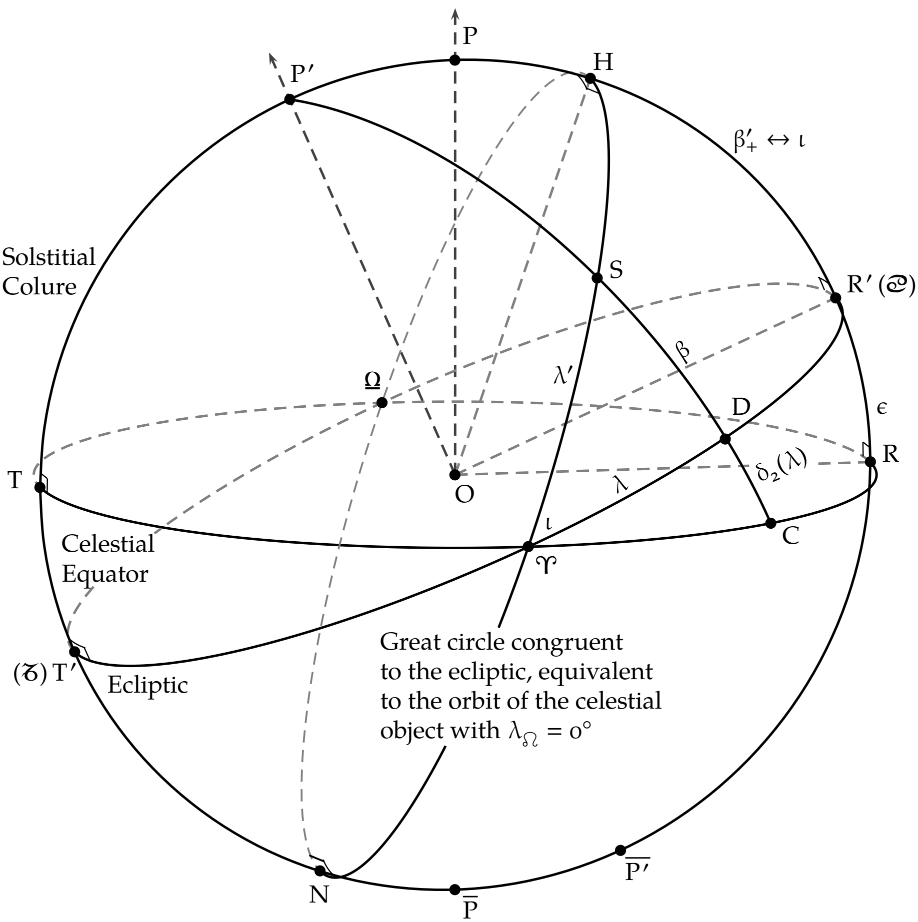
4.2.3 *Arcs of maximum true declination and maximum latitude*

In the *Sarvasiddhāntarāja* I.*spa·krā*, verses 5 and 6, Nityānanda defines the following two constituent arcs of the solstitial colure (*āyana-vṛtta*):

1. the arc of maximum true declination (*para-sphuṭa-apama*) that corresponds to the distance between the celestial equator (*viṣuva-vṛtta*) and the [great] circle congruent to the ecliptic (*bhavṛtta-sadṛśa-vṛtta*), and
2. the arc of maximum latitude (*para-iṣu*) that corresponds to the distance between the ecliptic (*bhavana-cakra*) and the [great] circle congruent to the ecliptic (*bhavṛtta-sadṛśa-vṛtta*).

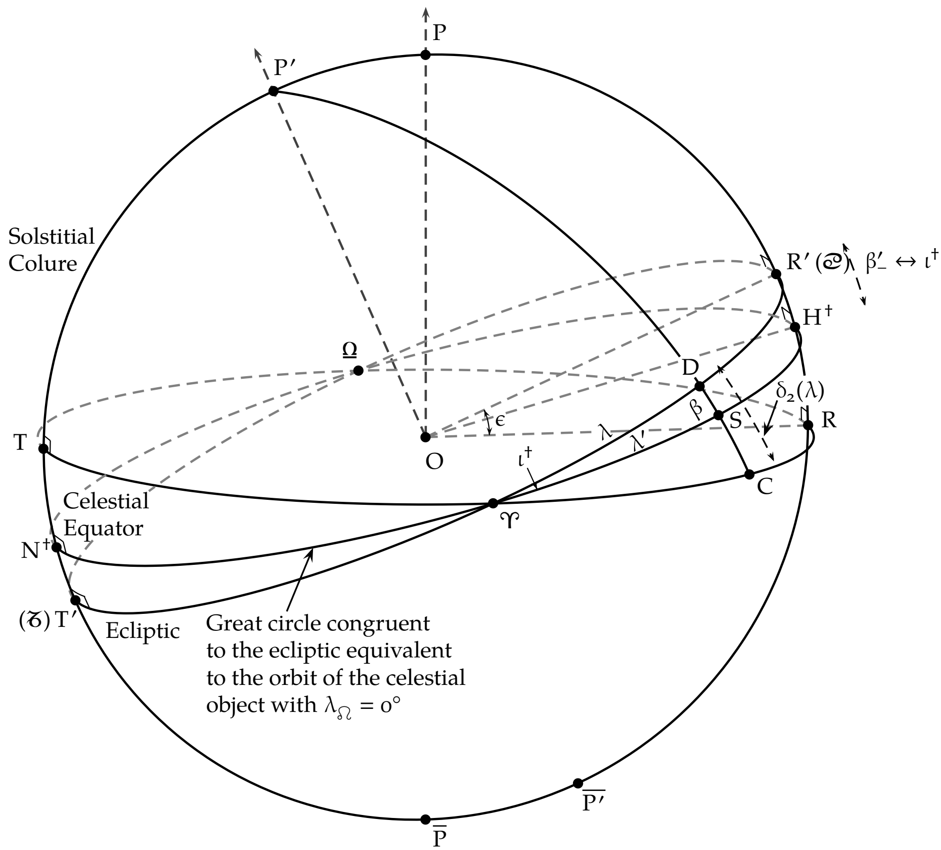
Figure 4a shows the great circle congruent to the ecliptic in a supra-ecliptic configuration similar to Figure 2a. Here, arcs $\widehat{\mathrm{RH}}$ and $\widehat{\mathrm{R'H}}$ represents the arcs of maximum true declination and maximum latitude respectively. The circle $\bigcirc\mathrm{N}\Upsilon\mathrm{H}\Omega$ represents the great circle congruent to the ecliptic that passes through the celestial object positioned at S at a given moment of time, and is inclined to the ecliptic at an arc angle of $\widehat{\mathrm{R'H}} = \beta'_+ = \iota$. In this configuration, the circle $\bigcirc\mathrm{N}\Upsilon\mathrm{H}\Omega$ also represents the orbit of the celestial object (with the longitude of the ascending node λ_{Ω} equal to zero). The arc $\widehat{\mathrm{RR'}}$ is the obliquity ϵ of the ecliptic to the celestial equator, seen here to be less than the arc of maximum true declination of the celestial object.

Similarly, Figure 4b represents a sub-ecliptic configuration of the great circle congruent to the elliptic comparable to Figure 2b. Here, the arcs $\widehat{\mathrm{RH}^\dagger}$ and $\widehat{\mathrm{R'H}^\dagger}$ represents the arcs of maximum true declination and maximum latitude respectively. The circle $\bigcirc\mathrm{N}^\dagger\Upsilon\mathrm{H}^\dagger\Omega$ represents the great circle congruent to the ecliptic that passes through the celestial object positioned at S at a given moment of time, and is inclined to the ecliptic at an arc angle of $\widehat{\mathrm{R'H}^\dagger} = \beta'_- = \iota^\dagger$. Here, the circle $\bigcirc\mathrm{N}^\dagger\Upsilon\mathrm{H}^\dagger\Omega$ represents the orbit of the celestial object (when the longitude of the ascending node λ_{Ω} is taken as zero). As before, the arc $\widehat{\mathrm{RR'}}$ is the obliquity ϵ of the ecliptic to the celestial equator, seen here to be more than the arc of maximum true declination of the celestial object.



curve of true declination	$\widehat{SC} = \widehat{DC} + \widehat{SD} = \delta_2(\lambda) + \beta$
arc of maximum latitude	$\widehat{R'H} = \beta'_+$
arc of maximum true declination	$\widehat{RH} = \widehat{RR'} + \widehat{R'H} = \epsilon + \beta'_+$

(a) *Supra-ecliptic configuration*



curve of true declination	$\widehat{SC} = \widehat{DC} - \widehat{SD} = \delta_2(\lambda) - \beta$
arc of maximum latitude	$\widehat{R'H^{\dagger}} = \beta'_{-}$
arc of maximum true declination	$\widehat{RH^{\dagger}} = \widehat{RR'} - \widehat{R'H^{\dagger}} = \epsilon - \beta'_{-}$

(b) *Sub-ecliptic configuration*

Figure 4: The arcs of the maximum true declination and the maximum latitude of a celestial object (at S) in two different configurations. In both configurations, the great circle congruent to the ecliptic is also taken to be the orbit of the celestial object with the longitude of the ascending node $\lambda_{☊} = 0°$.
(a) *Supra-ecliptic configuration* with the great circle congruent to the ecliptic ○N♈HΩ above the ecliptic ○♈R′ΩT′.
(b) *Sub-ecliptic configuration* with the great circle congruent to the ecliptic ○N†♈H†Ω below the ecliptic ○♈R′ΩT′.

Mullā Farīd, in his *Zīj-i Shāh Jahānī* Discourse II.6, passages [10] and [11], refers to the arc of maximum latitude ($\widehat{R'H}$ or $\widehat{R'H}^{\dagger}$ in Figure 4) as *qaws-i avval* 'first arc' and the arc of maximum true declination ($\widehat{RH}$ or $\widehat{RH}^{\dagger}$ in Figure 4) as *qaws-i duvum* 'second arc' (see Misra 2021: p. 90).

Remarks

1. In case of the Sun, the arc of maximum true declination is equal to the obliquity of the ecliptic, and hence, it remains constant. However, for some celestial objects like the Moon, their orbits precess about the ecliptic pole. In other words, the nodal line (*pāta*) of the orbit revolves in the plane of the ecliptic changing its nodal longitude (*pāta-bhāga*). This precession of the orbital nodes affects the value of the arc of maximum true declination by causing cyclical variations. For instance, in case of the Moon, the lunar orbit is inclined to the ecliptic (whose obliquity is 23.5°) at about 5.1°, and has a nodal precession of 18.6 years (with respect to the vernal equinox or 0° Aries). The draconic monthly variation in the maximum declination of the Moon changes from about ±18.4° at the minor lunar standstill to about ±28.6° at the major lunar standstill every 9.3 years. Nityānanda's use of the word *tatkṣane* 'at that very moment' in verse 5 suggests that he recognises the variation in the arc of maximum true declination (of certain celestial objects) with time.
2. For the arc of maximum latitude measured on the solstitial colure to correspond to the distance between the ecliptic and the great circle congruent to the ecliptic, one of the nodes (*pāta*) of the orbit of the celestial object needs to be coincident with the vernal equinoctial point ♈. In the last *pāda* of verse 6, Nityānanda explicitly states that the equinoctial point (*viṣuva*) and the orbital node (*pāta*) are in conjunction (*yuga*). In both configurations of Figure 4, the longitude of the ascending node $\lambda_{☊}$ is taken as 0°. Hence, the arc of maximum latitude $\widehat{R'H} = \beta'_{+} = \iota$ in Figure 4a or $\widehat{RH}^{\dagger} = \beta'_{-} = \iota^{\dagger}$ in Figure 4b. More generally,

$$\text{the arc of maximum latitude} = \beta'_{+/-} = \iota \text{ or } \iota^{\dagger}, \tag{14}$$

   depending on the orientation of the great circle congruent to the ecliptic (i.e., the orbit of the celestial object) and the ecliptic.
3. With the nodal longitude $\lambda_{☊/☋} = 0°$, the great circle congruent to the ecliptic can be regarded as the orbit of the celestial object, and hence,
   - for the supra-ecliptic configuration in Figure 4a, the arc of maximum true declination $\widehat{R'H}$ is the sum of the ecliptic obliquity $\widehat{RR'}$ and

the orbital inclination (or the arc of maximum latitude) $\widehat{R'H}$, i.e., $\delta_{true}^{max} = \epsilon + \iota$ or $\epsilon + \beta'_{+}$; and

- for the sub-ecliptic configuration in Figure 4b, the arc of maximum true declination $\widehat{R'H^{\dagger}}$ is the difference between the ecliptic obliquity $\widehat{RR'}$ and the orbital inclination (or the arc of maximum latitude) $\widehat{R'H^{\dagger}}$, i.e., $\delta_{true}^{max} = \epsilon - \iota^{\dagger}$ or $\epsilon - \beta'_{-}$.

Expressed more generally, we have the arc of maximum true declination

$$\delta_{true}^{max} = \epsilon + \iota \text{ or } \epsilon - \iota^{\dagger} \text{ or equivalently, } \epsilon \pm \beta'_{\pm}, \tag{15}$$

depending on the orientation of the great circle congruent to the ecliptic (i.e., the orbit of the celestial object) and the ecliptic.

4.2.4 Congruent arc and its complementary

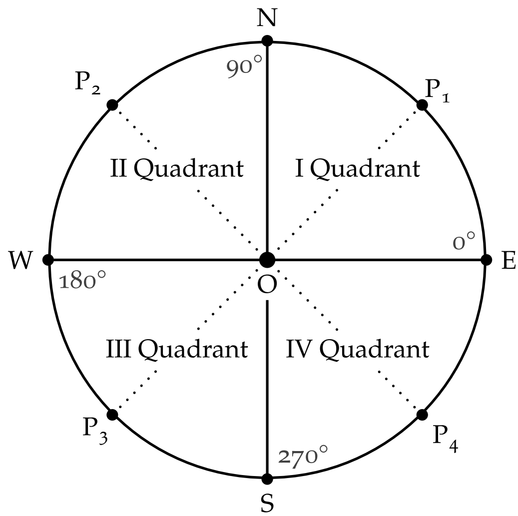
In the *Sarvasiddhāntarāja* I.*spa·krā*, verse 7, Nityānanda defines the following two constituent arcs of the circle congruent to the ecliptic (*bhavṛtta-sadṛśa-vṛtta*):

1. the congruent arc (*sadṛś-bhujā*) that measures the distance of the celestial object from the equinoctial point (*viṣuvat*), and
2. its complement, the congruent complementary arc (*sadṛś-koṭi*), that measures the distance of the celestial object from the solstitial colure (*āyana-vṛtta*).

In Figure 4a, $\widehat{\Upsilon S}$ is the congruent arc while $\widehat{SH} = 90° - \widehat{\Upsilon S}$ is the complement of the congruent arc. Similarly, in Figure 4b, $\widehat{\Upsilon S}$ is again the congruent arc while $\widehat{SH^{\dagger}} = 90° - \widehat{\Upsilon S}$ is the complement of the congruent arc.

The terms *bhujā* 'base' and *koṭi* 'complement of base' are often used in Sanskrit mathematics in connection to arcs of a circle (*cāpa*, *dhanus*, *kodaṇḍa*, etc.) or the chords/half-chords corresponding to arc (*jyā*, *jyakā*, *jyārdha*, etc.). Typically, the *bhujā* (also called *bhuja*, *bāhu*, or *dos*, synonyms for 'arm', 'side', or 'base') of an angle is calculated from (i) the degrees elapsed in odd quadrants of a circle, or (ii) the degree yet to be elapsed in even quadrants. The *koṭi* (sometimes understood as the 'perpendicular') refers to the complement of the *bhujā* in each quadrant. The *bhujā* and *koṭi* arcs in each of the four quadrants are shown below.

Quadrant	*bhujā*	*koṭi*
I	$\overparen{EP_1}$	$\overparen{P_1N}$
II	$\overparen{P_2W}$	$\overparen{NP_2}$
III	$\overparen{WP_3}$	$\overparen{P_3S}$
IV	$\overparen{P_4E}$	$\overparen{SP_4}$



For our present purpose, we distinguish between

1. the measures of *bhujā* and *koṭi* on the ecliptic, represented by arcs $\overparen{\Upsilon D}$ (longitude λ) and $\overparen{R'D}$ (co-longitude $\overline{\lambda}$) in Figure 1 respectively; and
2. the measures of *sadṛś-bhujā* and *sadṛś-koṭi* on the great circle congruent to the ecliptic, represented by arcs $\overparen{\Upsilon S}$ (congruent arc λ') and $\overparen{SH}$ (congruent complementary arc $\overline{\lambda'}$) in Figure 1 respectively.

As noted before (in § 4.1, p. 102), Mullā Farīd simply calls the longitude $\overparen{\Upsilon D}$ of the celestial object as *daraji-yi u* 'its degree' in his *Zīj-i Shāh Jahānī* Discourse II.6. The co-longitude $\overline{\lambda}$ of the celestial object is called *buᶜd-i daraji-yi kawkab az inqilāb-i aqrab* 'distance of the degree of a celestial object from the nearest solstice' in *Zīj-i Shāh Jahānī* Discourse II.6, passage [8]. In the same passage (and passage [9]), Mullā Farīd also calls the congruent complementary arc $\overparen{SH}$ as *buᶜd-i kawkab az «dāyiri-yi mārri bi aqṭāb-i arbaᶜi»* 'distance of a celestial object from the "circle passing through the four poles"' (see Misra 2021: p. 88).

4.3 PRELIMINARY COMPUTATIONS OF CONSTITUENT ARCS

4.3.1 *Computing the congruent arc and the congruent complementary arc*

Nityānanda proposes the following rules to compute the congruent arc (*sadṛkṣa-bāhu*) and congruent complementary arc (*sadṛkṣa-koṭi*) defined in § 4.2.4 in the *Sarvasiddhāntarāja* I.*spa·krā*, verses 8–9ab:

खगस्य कोटिसिञ्जिनी स्वबाणकोटिजीवया ॥
हता त्रिभज्यकोद्धृता सदृक्षकोटिसिञ्जिनी ॥ ८ ॥

तद्धनुर्नवति ९० तश्च्युतं यदा जायते सदृशबाहुसंज्ञकम् ॥ ९प्र,द्वि

khagasya koṭisiñjinī svabāṇakoṭijīvayā ||
hatā tribhajyakoddhṛtā sadṛkṣakoṭisiñjinī || 8 ||

taddhanur navati 90 taścyutaṃ yadā jāyate sadṛśabāhusaṃjñakam || 9ab

The Sine of the complement of the arc of longitude of a celestial object (*khagasya koṭi-siñjinī*), having been multiplied (*hatā*) by the Cosine of its latitude (*sva-bāṇa-koṭijīvā*) [and] having been divided (*uddhṛtā*) by the Radius [i.e., by the *sinus totus*] (*tribhajyakā*), [should be] the Sine of the congruent complementary arc (*sadṛkṣa-koṭi-siñjinī*) [i.e., the Sine of the complement of the *sadṛś-bhujā*]. 8

When [the measure of] its arc (*dhanus*), having been reduced from ninety [degrees] (*navatitaś-cyuta*), is determined, [it] has the name congruent arc (*sadṛśa-bāhu*). 9ab

In other words,

$$\mathrm{Sin}\left(\begin{array}{c}\text{congruent}\\ \text{complementary arc}\end{array}\right)=\frac{\mathrm{Sin}\left(\begin{array}{c}\text{complement of}\\ \text{arc of longitude}\end{array}\right)\cdot\mathrm{Cos}\,(\text{latitude})}{\textit{sinus totus}\ (\text{or Radius})},$$

from which,

$$\text{congruent arc} = 90^\circ - \mathrm{arcSin}\Big(\text{congruent complementary arc}\Big).$$

Expressed mathematically,

$$\mathrm{Sin}\,\overline{\lambda'} = \frac{\mathrm{Sin}\,\overline{\lambda}\cdot\mathrm{Cos}\,\beta}{\mathcal{R}} \Rightarrow \lambda' = 90^\circ - \mathrm{arcSin}\left(\mathrm{Sin}\,\overline{\lambda'}\right).$$

Looking at Figure 5, we can identify

- $\widehat{\Upsilon S} = \lambda'$ as the congruent arc (*sadṛkṣa-bāhu*), and hence $\widehat{SH} = 90^\circ - \lambda'$ or $\overline{\lambda'}$ as its complement, i.e., the congruent complementary arc (*sadṛkṣa-koṭi*);
- $\widehat{\Upsilon D} = \lambda$ as the longitude (*bhuja* or *bāhu*), and hence $\widehat{R'D} = 90^\circ - \lambda$ or $\overline{\lambda}$ as its complement, i.e., the co-longitude (*koṭi*); and
- $\widehat{SD} = \beta$ as the latitude (*bāṇa*), and hence $\widehat{P'S} = 90^\circ - \beta$ or $\overline{\beta}$ as its complement, i.e., the co-latitude.

By applying the Rule of Four Quantities to the right spherical triangles $\triangle P'HS$ and $\triangle P'R'D$ we find,[33]

33 See Van Brummelen (2013: 59–60) for the proof of Abū Naṣr's Rule of Four Quantities applied to right spherical triangles.

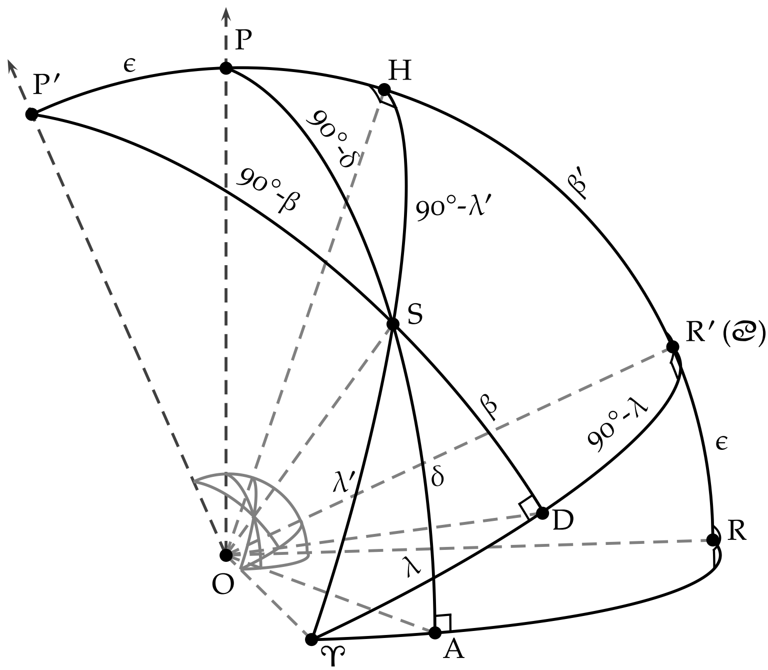


Figure 5: Right spherical triangles △P′HS and △P′R′D on the celestial sphere.

$$\frac{\sin \widehat{SH}}{\sin \widehat{R'D}} = \frac{\sin \widehat{P'S}}{\sin \widehat{P'D}} \text{ or equivalently, } \frac{\sin(90^\circ - \lambda')}{\sin(90^\circ - \lambda)} = \frac{\sin(90^\circ - \beta)}{\sin 90^\circ}.$$

Thus, $\sin(90^\circ - \lambda') = \sin(90^\circ - \lambda) \cdot \sin(90^\circ - \beta)$ or

$$\sin \overline{\lambda'} = \sin \overline{\lambda} \cdot \cos \beta. \tag{16}$$

For a non-unitary Radius (*sinus totus*), we then have

$$\operatorname{Sin} \overline{\lambda'} = \frac{\operatorname{Sin} \overline{\lambda} \cdot \operatorname{Cos} \beta}{\mathcal{R}}; \tag{17}$$

or in other words, Sin (*sadṛkṣa-koṭi*) $= \dfrac{\text{Sin}\,(\textit{koṭi}) \cdot \text{Cos}\,(\textit{bāṇa})}{\mathcal{R}}$ agreeing with verse 8.

With Sin (*sadṛkṣa-koṭi*) in equation (17), Nityānanda states, in verse 9ab, that the congruent arc or *sadṛkṣa-bāhu* is 90° − arcSin [Sin (*sadṛkṣa-koṭi*)], or

$$\lambda' = 90^\circ - \operatorname{arcSin}\left(\operatorname{Sin} \overline{\lambda'}\right), \text{ i.e.,}$$

$$\lambda' = 90° - \operatorname{arcSin}\left(\frac{\operatorname{Sin}\overline{\lambda}\cdot\operatorname{Cos}\beta}{\mathcal{R}}\right). \tag{18}$$

The Sine of the congruent arc, i.e., Sin (*sadṛkṣa-bāhu*) or Sin (λ'), can then be readily computed from the *sadṛkṣa-bāhu* or λ'.

Remarks

1. In Mahendra Sūri's method to compute the true declination of a celestial object in his *Yantrarāja* I.41–48, equation (17) is expressed as

$$\operatorname{Cos}\lambda' = \frac{\operatorname{Cos}\lambda\cdot\operatorname{Cos}\beta}{\mathcal{R}}.$$

The identification Sin $\overline{\lambda'}$ (Sine of the *sadṛkṣa-koṭi*) with Cos λ' (Cosine of the *sadṛkṣa-bāhu*) follows by recognising $\overline{\lambda'} = 90° - \lambda'$. Similarly, Sin $\overline{\lambda}$ (Sine of the *koṭi*) is identified with Cos λ (Cosine of the *bāhu*) with $\overline{\lambda} = 90° - \lambda$ (see Plofker 2000: equation 3 on p. 41 and equation 8 on p. 42).

2. Ibn Yūnis calls the Sin of equation (18) as *al-jayb al-awwal* 'first sine' in his derivation of the third method to compute the true declination in his *al-Zīj al-Kabīr al-Ḥākimī*, XXXIX.1.c (see King 1972: equation 1.10 on p. 295).

4.3.2 *Computing the maximum latitude and the maximum true declination*

In the *Sarvasiddhāntarāja* I.*spa·krā*, verses 9cd–10a, Nityānanda outlines the method of computing the arc of maximum latitude (*para-iṣu*), defined previously in § 4.2.3 as β', as follows:

या नभोगविशिखस्य सिञ्जिनी भाजिता ऽधरसदृक्षदोर्ज्यया ॥ ९त्रि,च ॥
तद्धनुः परशराह्वयो भवेद्... (१०प्र)

yā nabhogaviśikhasya siñjinī bhājitā 'dharasadṛkṣadorjyayā || 9cd ||
taddhanuḥ paraśarāhvayo bhaved... (10a)

Or, what is the Sine of the latitude of a celestial object (*nabhoga-viśikhasya siñjinī*), having been divided (*bhājitā*) by the lowered Sine of the congruent arc (*adhara-sadṛkṣa-dorjyā*), 9cd

its arc (*dhanus*) should be [called] the maximum latitude (*para-śara*) by name. ... (10a)

In other words,

$$\text{maximum latitude} = \operatorname{arcSin}\left[\frac{\operatorname{Sin}\,(\text{latitude})}{\operatorname{Sin}\,\big(\text{congruent arc}\big)/\textit{sinus totus}\ (\text{or Radius})}\right],$$

or expressed mathematically, $\beta' = \text{arcSin}\left(\dfrac{\text{Sin}\,\beta}{\text{Sin}\,\lambda'/\mathcal{R}}\right)$.

Looking at the right spherical triangles $\triangle\Upsilon\text{DS}$ and $\triangle\Upsilon\text{R}'\text{H}$ in Figure 5, we observe

- $\widehat{\Upsilon\text{S}} = \lambda'$ is the congruent arc (*sadṛkṣa-dos*) and $\widehat{\text{SD}} = \beta$ is the latitude (*viśikha*);
- $\widehat{\Upsilon\text{H}} = 90°$ and $\widehat{\text{HR}'} = \beta'$, the maximum latitude (*para-śara*).

Again, applying the Rule of Four Quantities to these right spherical triangles, we find

$$\frac{\sin\widehat{\text{HR}}}{\sin\widehat{\text{SD}}} = \frac{\sin\widehat{\Upsilon\text{H}}}{\sin\widehat{\Upsilon\text{S}}} \text{ or equivalently } \frac{\sin\beta'}{\sin\beta} = \frac{\sin 90°}{\sin\lambda'}.$$

Thus,

$$\sin\beta' = \frac{\sin\beta}{\sin\lambda'}. \tag{19}$$

For a non-unitary Radius (*sinus totus*), we then have

$$\text{Sin}\,\beta' = \frac{\text{Sin}\,\beta\cdot\mathcal{R}}{\text{Sin}\,\lambda'} \text{ or } \frac{\text{Sin}\,\beta}{\text{Sin}\,\lambda'/\mathcal{R}}, \tag{20}$$

$$\text{or effectively, } \beta' = \text{arcSin}\left(\frac{\text{Sin}\,\beta}{\text{Sin}\,\lambda'/\mathcal{R}}\right). \tag{21}$$

This is the expression *para-śara* = *dhanus* of $\left[\dfrac{\text{Sin}\,(\textit{bāṇa})}{\text{Sin}\,(\textit{adhara-sadṛkṣa-dos})}\right]$ in verses 9cd–10a.

Remarks

1. The compound *adhara-sadṛkṣa-dor-jyayā* 'by the lowered Sine (*adhara-jyā*) of the congruent arc (*sadṛkṣa-dos*)' in verse 9d refers to the divisor Sin $\lambda'/\mathcal{R}$. Nityānanda translates the Islamicate arithmetical operation of lowering (*munḥaṭṭ kardan*), i.e., dividing a quantity by sixty (equal to the Radius or the *sinus totus* $\mathcal{R}$), as *adhara-kṛ* (or *adharī-kṛ*) 'to lower'.
   MS Bn.II parses the word *adhara* in the middle of the verse on line 11 of f. 63r as …*adhara-sa-ṣaṣṭi-bhakta-bhājaka-bhajanam atrādhara-bhajana-saṃjñam* [*u*]*cyate* '…*adhara* is the sixtieth part [lit. with sixty divided] of the divisor of the division; here what is called *adhara*-division is declared'. Mullā Farīd, in his *Zīj-i Shāh Jahānī* Discourse II.6, passage [9] uses the term *munḥaṭṭ-i qismat kardan* 'to low-divide' while describing the same method of computing the arc of maximum latitude (which he calls *qaws-i avval* 'the first arc')

(see Misra 2021: 88). MS Bn.II is unique in parsing the meaning of the word *adhara* mid-verse, and also identifying the operation as *adhara-bhajana* 'low-division'.

2. In verse 10a, Nityānanda uses the word *taddhanus* 'the arc of that value' to refer to the measure of arc corresponding to the result of the previous division (in verse 9cd). This suggests that the division $\dfrac{\mathrm{Sin}\,\beta}{\mathrm{Sin}\,\lambda'/\mathcal{R}}$ in equation (20) yields a measures equivalent to the Sine of a quantity, which, in this case, is Sin (β'). Therefore, the arc (*dhanus*) of maximum latitude (*para-śara*) is the inverse Sine of the result of the division as equation (18) shows. Nityānanda's expression for the maximum latitude differs from what Mahendra Sūri proposes in his *Yantrarāja* I.41–48. As Plofker describes, Mahendra Sūri's method takes $\beta' = \mathrm{Sin}\,\beta/\mathrm{Sin}\,\lambda'$. The result of this division is called *bhāgādikam* 'result in degrees etc.' (*Yantrarāja*: 1.42) that becomes *syād antaram* 'the difference', i.e., the additive correction, employed in the next step of this procedure. The omission of the factor of $\mathcal{R}$ and the approximation of Sin (β') as β' in Mahendra Sūri's method appear to be possible early corruptions or misinterpretations (see Plofker 2000: 42–43).
3. In his *al-Zīj al-Kabīr al-Ḥākimī*, XXXIX.1.c, Ibn Yūnis calls the arc of maximum latitude *al-qaws al-iṣlāḥ* 'the correction arc' (see King 1972: equation 1.11 on p. 295).

In the *Sarvasiddhāntarāja* I.*spa·krā*, verse 10bcd–11, Nityānanda describes the arc of maximum true declination (*para-sphuṭa-krānti*) δ^{max}_{true} (previously defined in § 4.2.3) as

वा परेषुपरमापमाख्ययोः ॥
संयुतिर्वियुतिरस्ति च क्रमाद्गोलबाणसमभिन्नदिक्तया ॥ १०द्वि,त्रि,च ॥

स ग्रहस्य परमस्फुटापमो जायते युतिवियोगदिक्स्थितः ॥
एवमभ्रनव ९० तो ऽधिको यदा खाष्टभू १८० परिमितेर्विशोधितः ॥ ११ ॥

vā pareṣuparamāpamākhyayoḥ ||
saṃyutir viyutir asti ca kramād golabāṇasamabhinnadik tayā || 10bcd ||

sa grahasya paramasphuṭāpamo jāyate yutiviyogadik sthitaḥ ||
evam abhranava 90 to 'dhiko yadā khāṣṭabhū 180 parimiter viśodhitaḥ || 11 ||

There is the sum (*saṃyuti*) or the difference (*viyuti*) of the two [quantities] known as the maximum latitude (*para-iṣu*) and the greatest declination (*parama-apama*) [i.e., the obliquity of the ecliptic] with the

> latitude (*bāṇa*) and the celestial hemisphere (*gola*) [i.e., the declination of the celestial object] in the same or different directions (*sama-bhinna-diś*) respectively. 10bcd
>
> That [result], being situated in the direction of the conjunction or the disjunction (*yuti-viyoga-diś*), becomes the maximum true declination of a celestial object (*grahasya parama-sphuṭa-apama*). Thus, when [its measure is] greater (*adhika*) than ninety [degrees] (*abhra-nava*), [it is] made to be subtracted (*viśodhita*) from a measure of one hundred and eighty [degrees] (*kha-aṣṭa-bhū*). 11

In other words,

$$\text{maximum true declination} = \text{ecliptic obliquity} \pm \text{maximum latitude},$$
$$\text{with } 0^\circ \leq \text{maximum true declination} \leq 90^\circ.$$

Expressed mathematically,

$$\delta_{true}^{max} = \epsilon \pm \beta'_{\pm} \text{ where } \delta_{true}^{max} = \left[180^\circ - \left(\epsilon + \beta'_{+}\right)\right] \ \forall \ \epsilon + \beta'_{+} > 90^\circ.$$

From equation (15), we known that $\delta_{true}^{max} = \epsilon + \beta'_{+}$ or $\delta_{true}^{max} = \epsilon - \beta'_{-}$, and hence,

$$\delta_{true}^{max} = \epsilon \pm \beta'_{\pm}. \tag{22}$$

The choice of addition or subtraction depends on the orientation of the great circle congruent to the ecliptic (understood here, as the orbit of the celestial object with the nodal longitude being zero, i.e., the node of the orbit coincident with the vernal equinoctial point ♈︎).

According to Nityānanda, when the latitude (*bāṇa*) and the celestial hemisphere (*gola*)—i.e., the direction of the celestial object in relation to the celestial equator; in other words, its declination—are similarly oriented, the arc of maximum true declination is the addition of the ecliptic obliquity (*parama-apama*, lit. greatest declination) and the arc of maximum latitude (*para-śara*). When the latitude and the celestial hemisphere are differently oriented, arc of maximum true declination is the difference between the ecliptic obliquity and the arc of maximum latitude.

Figure 6 shows the two configurations for the orbit of a celestial object. The node of the orbit is coincident with the vernal equinoctial point in both configurations, with the obliquity of the ecliptic $\widehat{TT'} = \widehat{RR'} = \epsilon$.

1. For the orbital path $N^{+}S_1^{+}S_2^{+}S_3^{+}S_4^{+}H^{+}$, the declination and the latitude of the celestial object are both directed towards the northern hemisphere of the celestial sphere, i.e., towards P and P′ respectively. These are seen at positions S_3^{+} and S_4^{+} of the figure. Alternatively, both the declination and the latitude of the celestial object are directed towards the southern hemisphere of the celestial sphere, i.e., towards $\overline{P}$ and $\overline{P'}$ respectively. These are the positions S_1^{+} and S_2^{+} in the figure. In both these cases, it can be seen that

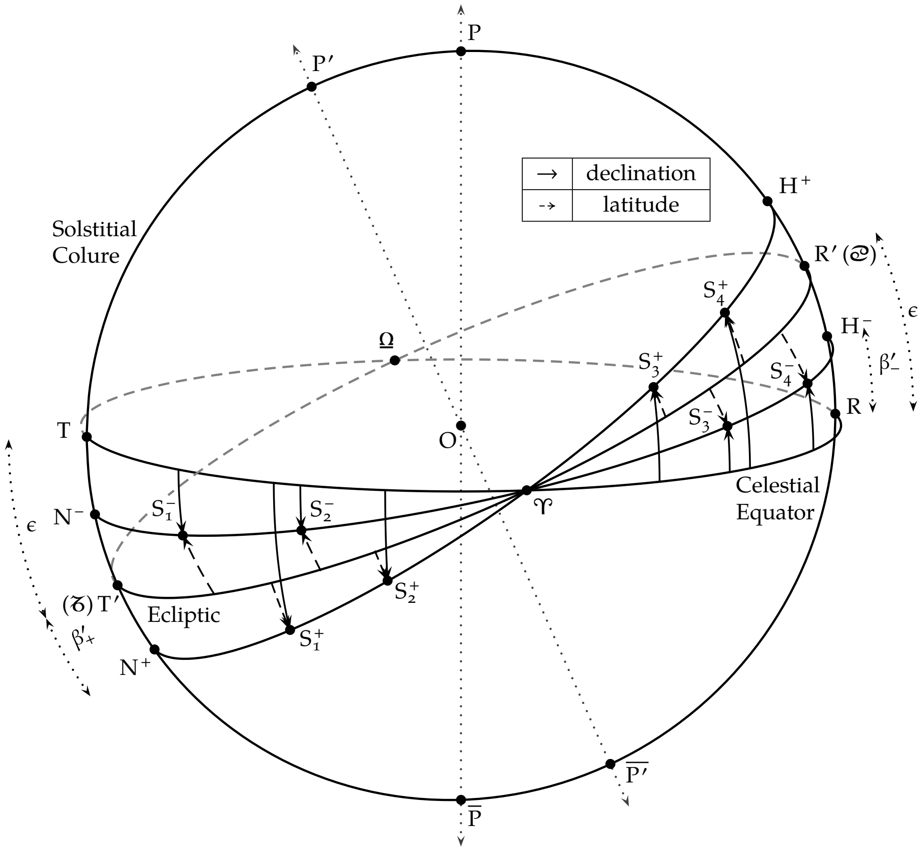


Figure 6: Configuration of the orbit of a celestial object with (a) its declination and latitude being similarly orientated in their respective hemispheres, i.e., both towards the northern or southern half of the celestial sphere, represented here by the path $N^+S_1^+S_2^+S_3^+S_4^+H^+$); or (b) its declination and latitude being oppositely oriented in their respective hemispheres, i.e., one towards the northern half of the celestial sphere and the other towards the southern half (or vice versa), represented here by the path $N^-S_1^-S_2^-S_3^-S_4^-H^-$.

- the maximum latitude $\widehat{N^+T'} = \widehat{H^+R'} = \beta'_+$, and
- the maximum true declination $\widehat{N^+T} = \widehat{H^+R} = \delta_{true}^{max} = \epsilon + \beta'_+$.

2. For the orbital path $N^-S_1^-S_2^-S_3^-S_4^-H^-$, the declination and the latitude of the celestial object are directed towards the northern (towards P) and southern (towards $\overline{P'}$) hemispheres of the celestial sphere respectively. Positions S_3^- and S_4^- in the figure depict this configuration. Reciprocally, positions S_1^+ and S_2^+ show the celestial object with its declination and the latitude directed

towards the southern (towards $\overline{\mathrm{P}}$) and northern (towards P′) hemispheres of the celestial sphere respectively. In both these cases, we find

- the maximum latitude $\widehat{\mathrm{N^-T'}} = \widehat{\mathrm{H^-R'}} = \beta'_-$, and
- the maximum true declination $\widehat{\mathrm{N^-T}} = \widehat{\mathrm{H^-R}} = \delta^{max}_{true} = \epsilon - \beta'_-$.

Remarks

1. In Mahendra Sūri's method in his *Yantrarāja* I.43, the greatest declination (i.e., the obliquity of the ecliptic) is increased or decreased by the maximum latitude (called the additive correction in degrees etc., see note 2 on p. 121) "when [the longitude] of the star and the latitude are in the same or different hemispheres respectively" (see Plofker 2000: 41).
2. In his *al-Zīj al-Kabīr al-Ḥākimī*, XXXIX.1.c, Ibn Yūnis calls the arc of maximum true declination *al-qaws al-māʾil* 'the inclination arc' *al-qaws al-iṣlāḥ* 'the correction arc' (see King 1972: equation 1.12 on p. 295).

The direction of the maximum true declination δ^{max}_{true} is along the direction of the sum (*yuti*) of the ecliptic obliquity and the maximum latitude, or along the residue (*viyuti*) of the maximum latitude removed from the ecliptic obliquity. The value of the maximum true declination lies between 0° and 90°. Figure 7 depicts the configuration where the great circle congruent to the ecliptic (i.e., the orbit of a celestial body) is inclined to the ecliptic at an angle larger than $90° - \epsilon$. In this case, as Nityānanda explains,

$$\delta^{max}_{true} = \epsilon + \beta'_+ \equiv \left[180° - (\epsilon + \beta'_+)\right] \ \forall \ \epsilon + \beta'_+ > 90°. \tag{23}$$

4.4 THE FIRST METHOD OF TRUE DECLINATION

Nityānanda's describes the first method to compute the true declination of a celestial object in *Sarvasiddhāntarāja* I.*spa·krā*, verse 2, as follows:

स्फुटापमाङ्कसिञ्जिनी सभत्रयद्युजीवया ॥
हता त्रिभज्यकोद्धृता स्फुटापमज्यका भवेत् ॥ २ ॥

sphuṭāpamāṅkasiñjinī sabhatrayadyujīvayā ||
hatā tribhajyakoddhṛtā sphuṭāpamajyakā bhavet || 2 ||

The Sine of the curve of true declination (*sphuṭa-apama-aṅka-siñjinī*), having been multiplied (*hatā*) by the day-Sine [of the longitude] increased by three zodiacal signs (*sa-bha-traya-dyujīvā*) [i.e., by the Cosine of the first declination of the 'longitude increased by 90°'] [and] having been divided (*uddhṛtā*) by the Radius [i.e., by the *sinus totus*]

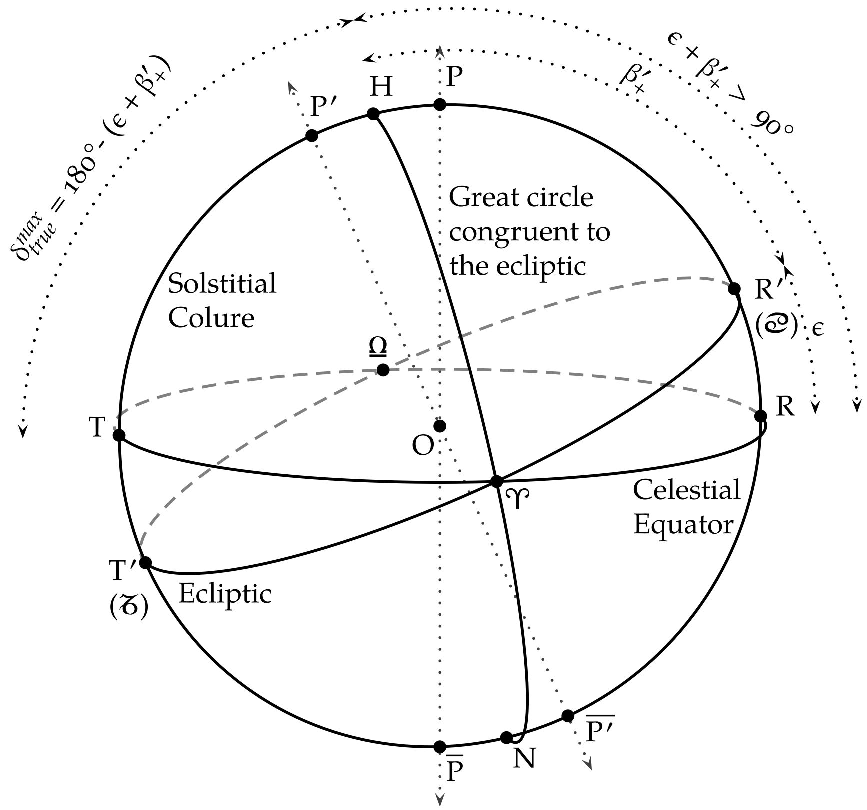


Figure 7: Configuration of the great circle congruent to the ecliptic (i.e., the orbit of a celestial object) in the celestial sphere where $\epsilon + \beta'_+ > 90°$.

> (*tribhajyakā*) should be the Sine of the true declination (*sphuṭa-apama-jyakā*). 2

In other words,

$$\text{Sin}\begin{pmatrix}\text{true}\\\text{declination}\end{pmatrix} = \frac{\text{Sin}\begin{pmatrix}\text{curve of}\\\text{true declination}\end{pmatrix}\cdot\text{Cos}\begin{pmatrix}\text{first declination}\\\text{of the longitude}\\\text{increased by } 90°\end{pmatrix}}{\textit{sinus totus}\text{ (or Radius)}},$$

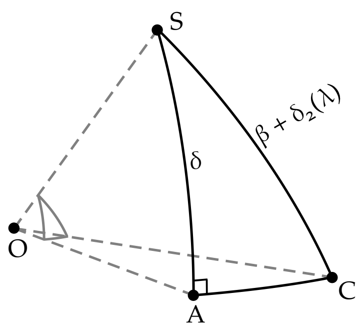
or expressed mathematically, $\text{Sin}\,\delta = \dfrac{\text{Sin}\left[\delta_2(\lambda) \pm \beta\right]\cdot\text{Cos}\,\delta_1(90° + \lambda)}{\mathcal{R}}$ where

Mathematical expression	*Sanskrit expression*	*English translation*
Sin δ	*sphuṭa-apama-jyakā*	Sine of the true declination,
Sin $\left[\delta_2(\lambda) \pm \beta\right]$	*sphuṭa-apama-aṅka-siñjini*	Sine of the curve of true declination,
Cos $\delta_1(90° + \lambda)$	*sa-bha-traya-dyujīvā*	day-Sine [of the longitude] increased by three zodiacal signs, i.e., Cosine of the first declination of the 'longitude increased by 90°', and
$\mathcal{R}$	*tribhajyakā*	Radius or *sinus totus*.

4.4.1 *Derivation of the first method*

The mathematical expression of the first method of true declination can be derived as follows:

1. The excerpt (from Figure 1) below, to the left, shows, the right spherical triangle $\triangle$SAC has $\widehat{SC} = \beta + \delta_2(\lambda)$, or more generally, $\delta_2(\lambda) \pm \beta$ (see equation (11)), $\widehat{SA} = \delta$, and $\sphericalangle SAC = 90°$. Applying the spherical law of sines to triangle $\triangle$SAC gives



Excerpt of $\triangle$SAC from Figure 1.

$$\frac{\sin \widehat{SC}}{\sin \sphericalangle SAC} = \frac{\sin \widehat{SA}}{\sin \sphericalangle SCA}, \text{ i.e.,}$$

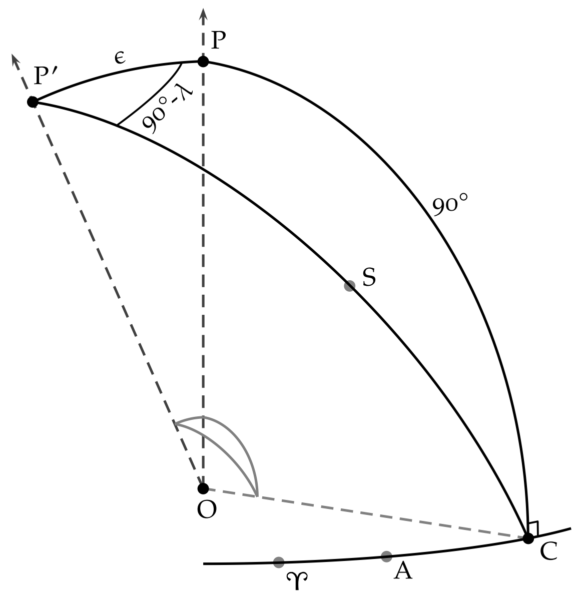
$$\frac{\sin\left[\delta_2(\lambda) \pm \beta\right]}{\sin 90°} = \frac{\sin \delta}{\sin \sphericalangle SCA} \text{ or}$$

$$\sin \delta = \sin\left[\delta_2(\lambda) \pm \beta\right] \cdot \sin \sphericalangle SCA. \quad (24)$$

2. Looking at the excerpt (from Figure 1) below, we can recognise that $\widehat{P'P} = \epsilon$, $\sin \sphericalangle PP'C = 90° - \lambda$ since $\sphericalangle PP'C$ is the angle of $\widehat{R'D}$, and $\sin \widehat{PC} = 90°$ as P is the pole with respect to point C on the celestial equator. Again, the spherical law of sines applied to the spherical triangle $\triangle$ PP′C gives

$$\frac{\sin \widehat{PC}}{\sin \sphericalangle PP'C} = \frac{\sin \widehat{P'P}}{\sin \sphericalangle P'CP}, \text{ i.e., } \sin \sphericalangle P'CP = \frac{\sin \widehat{P'P} \cdot \sin \sphericalangle PP'C}{\sin \widehat{PC}} \text{ or}$$

$$\sin \sphericalangle P'CP = \frac{\sin \epsilon \cdot \sin (90^\circ - \lambda)}{\sin 90^\circ}. \tag{25}$$



Excerpt of $\triangle P'PC$ from Figure 1.

3. From equation (25), we have,

$$\sin \sphericalangle P'CP = \sin \epsilon \cdot \cos \lambda \Rightarrow \sin \sphericalangle P'CP = \sin \epsilon \cdot \sin (90^\circ + \lambda). \tag{26}$$

4. The 'first declination' of a celestial object with, say, longitude x° can be given by the 'method of declination' as $\sin [\delta_1(x^\circ)] = \sin \epsilon \cdot \sin x^\circ$, see equation (9).
5. Thus, equation (26) can be expressed as

$$\sin \sphericalangle P'CP = \sin \delta_1(90^\circ + \lambda) \Rightarrow \sphericalangle P'CP = \delta_1(90^\circ + \lambda).\text{[34]} \tag{27}$$

6. We can also recognise that $\sphericalangle PC♈$ is a right angle, and hence,

$$\begin{aligned} &\sphericalangle P'C♈ = \sphericalangle PC♈ - \sphericalangle PCP' \Rightarrow \sphericalangle P'C♈ = 90^\circ - \delta_1(90^\circ + \lambda) \\ \Rightarrow\ &\sphericalangle P'C♈ = \sphericalangle SCA = 90^\circ - \delta_1(90^\circ + \lambda). \end{aligned} \tag{28}$$

34 In Islamicate astronomy, the quantity $\delta_1(90^\circ \pm \lambda)$ is called the 'inverse declination' (*al-māyl al-maʿkūs*) of a point [with standard ecliptic coordinates (β, λ)], e.g., in the late twelfth-century *zījes* of Marāgha astronomers like the *Zīj-i Ilkhānī* of al-Ṭūsī (see Hamadani-Zadeh 1987: 188) and *Tāj al-Azyāj* of Muḥyī l-Dīn al-Maghribī (see Dorce 2002–3: 196).

7. From equations (24) & (28), we have

$$\sin\delta = \sin\left[\delta_2(\lambda) \pm \beta\right] \cdot \sin\left[90^\circ - \delta_1(90^\circ + \lambda)\right] \text{ or equivalently,}$$
$$\sin\delta = \sin\left[\delta_2(\lambda) \pm \beta\right] \cdot \cos\delta_1(90^\circ + \lambda). \tag{29}$$

For a non-unitary Radius (*sinus totus*), we then have

$$\operatorname{Sin}\delta = \frac{\operatorname{Sin}\left[\delta_2(\lambda) \pm \beta\right] \cdot \operatorname{Cos}\delta_1(90^\circ + \lambda)}{\mathcal{R}}. \tag{30}$$

4.4.2 *Historical testimonies of the first method*

The first method of true declination, expressed mathematically in equation (30), can be found in several Islamicate works (prior to Mullā Farīd's *Zīj-i Shāh Jahānī*). For example,

1. The letter (№ 2) of Abū Naṣr Manṣūr b. ᶜAlī b. ᶜIrāq (960–c. 1036) to al-Bīrūnī on the proof of the operations in the 'table of rectifications' in the *zīj* of Ḥabash al-Ḥāsib (766–d. c. *post* 869) called *Risālah fī barāhīn aᶜmāl Ḥabash bi-jadwal al-taqwīm*.[35] Irani (1956: 90–91) discusses Abū Naṣr Manṣūr b. ᶜAlī b. ᶜIrāq's proof of Ḥabash al-Ḥāsib's operation for finding the declination of a star by the table of rectification, identical to Nityānanda's rule in equation (30).
2. Also, Abū Naṣr Manṣūr b. ᶜAlī b. ᶜIrāq's letter (№ 9) to al-Bīrūnī, entitled the *Risāla fī maᶜrifat al-qusīy al-fallakiyya bi-ṭārīq ghair ṭarīq al-nisba al-muᶜallafa*, describes a method (different to the method of compounded ratios) to compute the various arcs on the sphere; one of the steps in Abū Naṣr Manṣūr b. ᶜAlī b. ᶜIrāq's method is identical to equation (30).[36] See Luckey (1941: 445) for a technical study of Abū Naṣr Manṣūr b. ᶜAlī b. ᶜIrāq's rules; in particular, see pages 430 (line 17) to 431 (line 28) for a German translation of the rule to compute the arc of declination based on the Arabic MS 2468/2519 from the Bankipore collection of the Khuda Bakhsh Oriental Library in Patna (India).
3. The rule of equation (30) is also seen in Abu ᵓl-Wafā Būzhjānī's *Kitāb al-Majisṭī* (c. 997–c. 1010) read from MS Paris BnF 2494 «*L'Almageste d'Abou ᵓl-Wafâ al-Boûzdjânî*», ff. 67v:18–69v:17, held at the *Bibliothèque nationale de France* in Paris [37] (see Debarnot 1985: footnote 3 on p. 212).

35 Recorded as Tract № 4, 71 pages, in the collection of his letters *Rasāᶜil ᵓl-Bīrūnī* published by Osmaniya Oriental Publications Bureau, Hyderabad 1948.

36 Recorded as Tract № 8, 13 pages, in the *Rasāᶜil ᵓl-Bīrūnī*, op. cit. reference in footnote 35.

37 Open access via *Bibliothèque nationale de France* Digital Library *Gallica* at `ark:/12148/btv1b100374763`.

4. al-Bīrūnī's *Kitāb Taḥdīd al-Amākin* (c. eleventh century) Chapter V.63 states the (first) rule to compute the true declination of a star that is identical to equation (30) (see Kennedy 1973: 119–121).
5. al-Ṭūsī's (first) method to find the declination of 'other points' using the inverse in his *Zīj-i Ilkhānī* (c. late thirteenth century) is identical to equation (30) (see Hamadani-Zadeh 1987: 188). Hamadani-Zadeh describes al-Ṭūsī's instructions on calculating the inverse declination of the point (with longitude λ); however, he incorrectly calls 'the first declination δ_1 corresponding to λ' as the inverse declination on p. 188, lines 10–11. The inverse declination is correctly understood as $\delta_1(90° + \lambda)$.
6. The *Zīj-i Jadīd-i Sulṭānī* Discourse II.5 (alias *Zīj-i Ulugh Beg*, published 1438–1439) of Sulṭān Ulugh Beg describes three methods to determine the actual distance (i.e., true declination) of a star from the (celestial) equator. The first method, expressed mathematically, is identical to equation (30) (see Sédillot 1853: 89). The description of the first method also states how the second declination and latitude are to be added when they have the same sign (with respect to the equator); otherwise they are to subtracted. Mullā Farīd's *Zīj-i Shāh Jahānī* Discourse II.6, repeats these statements from Ulugh Beg's *Zīj-i Jadīd-i Sulṭānī* Discourse II.5 near-verbatim.

Among contemporaneous Sanskrit authors, Nityānanda's first method of true declination also appears in Munīśvara's *Siddhāntasārvabhauma* I.4.41–42 (1646) and Kamalākara's *Siddhāntatattvaviveka* VIII.23cd–24ab (1658). Appendices D and E include the Sanskrit text, English translations, and technical discussions of Munīśvara's rule and Kamalākara's first method respectively.

4.5 THE SECOND METHOD OF TRUE DECLINATION

Nityānanda's describes the second method to compute the true declination of a celestial object in *Sarvasiddhāntarāja* I.*spa·krā*, verse 3, as follows:

> परमक्रान्तिकोटिज्या स्फुटक्रान्त्यङ्कजीवया ॥
> हतान्यक्रान्तिकोटिज्याप्ता स्यात्स्पष्टापमज्यका ॥ ३ ॥
>
> *paramakrāntikoṭijyā sphuṭakrāntyaṅkajīvayā ||*
> *hatānyakrāntikoṭijyāptā syāt spaṣṭāpamajyakā || 3 ||*
>
> The Cosine of the greatest declination (*parama-krānti-koṭijyā*) [i.e., the Cosine of the ecliptic obliquity], having been multiplied (*hatā*) by the Sine of the curve of true declination (*sphuṭa-krānti-aṅka-jīvā*) [and] having been divided (*āptā*) by the Cosine of the other declination (*anya-krānti-koṭijyā*) [i.e., by the Cosine of the second declination], should be the Sine of the true declination (*spaṣṭa-apama-jyakā*). 3

In other words,

$$\text{Sin}\left(\begin{array}{c}\text{true}\\ \text{declination}\end{array}\right)=\frac{\text{Cos}\left(\begin{array}{c}\text{ecliptic}\\ \text{obliquity}\end{array}\right)\cdot\text{Sin}\left(\begin{array}{c}\text{curve of}\\ \text{true declination}\end{array}\right)}{\text{Cos}\ (\text{second declination})},$$

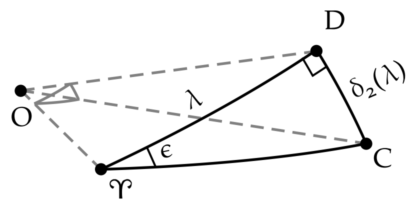
or expressed mathematically, $\text{Sin}\,\delta=\dfrac{\text{Cos}\,\epsilon\cdot\text{Sin}\left[\delta_2(\lambda)\pm\beta\right]}{\text{Cos}\,\delta_2(\lambda)}$ where

Mathematical expression	*Sanskrit expression*	*English translation*
$\text{Sin}\,\delta$	*spaṣṭa-apama-jyakā*	Sine of the true declination,
$\text{Cos}\,\epsilon$	*parama-krānti-koṭijyā*	Cosine of the greatest declination, i.e., Cosine of the ecliptic obliquity,
$\text{Sin}\left[\delta_2(\lambda)\pm\beta\right]$	*sphuṭa-krānti-aṅka-jīvā*	Sine of the curve of true declination, and
$\text{Cos}\,\delta_2(\lambda)$	*anya-krānti-koṭijyā*	Cosine of the other declination, i.e., the Cosine of the second declination.

4.5.1 *Derivation of the second method*

The mathematical expression of the second method of true declination can be derived following King (1972: pp. 293–295) where a similar expression from Ibn Yūnis's *al-Zīj al-Kabīr al-Ḥākimī*: 39.1(b) is explained.

1. In the right spherical triangle $\triangle\Upsilon DC$, shown below, to the left, we observe, $\sphericalangle CD\Upsilon = 90°$, $\sphericalangle D\Upsilon C = \overset{\frown}{R'R} = \epsilon$, and $\overset{\frown}{DC} = \delta_2(\lambda)$. Applying Geber's theorem to the right spherical triangle $\triangle\Upsilon DC$, we have,[38]



Excerpt of $\triangle\Upsilon DC$ from Figure 1.

$$\cos\sphericalangle D\Upsilon C=\cos\overset{\frown}{DC}\cdot\sin\sphericalangle DC\Upsilon \text{ or}$$
$$\cos\epsilon=\cos\delta_2(\lambda)\cdot\sin\sphericalangle DC\Upsilon, \text{ or}$$
$$\sin\sphericalangle DC\Upsilon=\frac{\cos\epsilon}{\cos\delta_2(\lambda)}. \tag{31}$$

38 See Van Brummelen (2013: 77–79) for the proof of Geber's theorem for right spherical triangles.

2. This allows us to write

$$\sin\delta = \sin\left[\delta_2(\lambda) \pm \beta\right] \cdot \sin\sphericalangle SCA \text{ from equation (24) as}$$
$$\sin\delta = \sin\left[\delta_2(\lambda) \pm \beta\right] \cdot \sin\sphericalangle DC\Upsilon \ \because \sphericalangle SCA = \sphericalangle DC\Upsilon \text{, see Figure 1.}$$
$$\text{Therefore, } \sin\delta = \sin\left[\delta_2(\lambda) \pm \beta\right] \frac{\cos\epsilon}{\cos\delta_2(\lambda).} \tag{32}$$

In terms of a non-unitary Radius (*sinus totus*), we then have

$$\mathrm{Sin}\,\delta = \frac{\mathrm{Cos}\,\epsilon \cdot \mathrm{Sin}\left[\delta_2(\lambda) \pm \beta\right]}{\mathrm{Cos}\,\delta_2(\lambda)}. \tag{33}$$

The equivalence between the mathematical formulae in verses 2 and 3, i.e., equations (30) and (33), is derived in Appendix C.

4.5.2 Historical testimonies of the second method

The second method of true declination, expressed mathematically in equation (33), can also be found in various Islamicate works (prior to Mullā Farīd's *Zīj-i Shāh Jahānī*).[39] For example,

1. the (first) method to determine the distance of a star from the equator in al-Bīrūnī's *Kitāb Maqālīd ʿilm al-Hayʾa* (c. 994) (see Debarnot 1985: 210–212);
2. the second method of determining the declination of a celestial body in Ibn Yūnis' *al-Zīj al-Kabīr al-Ḥākimī* (1003) (see King 1972: 39.1(b) on pp. 293–295);
3. the (second) method to find the declination of 'other points' in al-Ṭūsī's *Zīj-i Ilkhānī* (c. late thirteenth century) (see Hamadani-Zadeh 1987: 188);
4. the rule in chapters 1 and 2 of al-Kāshī's *Zīj al-Khāqānī* (c. 1413/1414), Treatise IV (see Kennedy 1985: 9) where the rule appear on f. 168r: 11–12 with its proof on f. 174v: 10–11 of MS London India Office Persian 430 (Ethé 2232) of the *Zīj al-Khāqānī*; and
5. the second method in Ulugh Beg's *Zīj-i Jadīd-i Sulṭānī* Discourse II.5 (published 1438–1439) (see Sédillot 1853: 90), and repeated near-verbatim in Mullā Farīd's *Zīj-i Shāh Jahānī* Discourse II.6.

39 One of the earlier statements of this rule can be found in Ptolemy's *Almagest* (c. second century CE), Book VIII.5 on the 'computation of simultaneous culmination of sun and star' (see Toomer 1984: 411). Ptolemy's method expresses this result in chords (instead of sines) and is derived from the first form of Menelaus' Proposition III.1 from his *Sphaerica* (see Neugebauer 1975: Theorem I on p. 28).

Remarks

1. The second method in Ulugh Beg's *Zīj-i Jadīd-i Sulṭānī* elaborates on the values of the true declination δ of a celestial object when its latitude β and first/second declination $\delta_{1/2}(\lambda)$ take particular values depending on its position in the sky.
   (a) When $\beta = 0°$, $\delta = \delta_1(\lambda)$. Here, the celestial object lies on the ecliptic (with no latitude) and hence, its true declination simply corresponds to the first declination (like, e.g., the declination of the Sun). Thus,

   $$\operatorname{Sin}\delta = \operatorname{Sin}\delta_1(\lambda) = \frac{\operatorname{Sin}\epsilon \cdot \operatorname{Sin}\lambda}{\mathcal{R}},$$

   using equation (9) for non-unitary Radius (*sinus totus*).
   (b) When $\beta \neq 0°$ and $\delta_1(\lambda) = 0°$, in other words, when the celestial object has a non-zero-latitude and its first declination is zero, the object lies on the equinoctial colure. The equinoctial colure is a great circle passing through the celestial poles and the equinoctial points; conceived in Figure 1 as a circle passing through the points ♈, P, and ♎. In this case, the second declination of the object is also zero, i.e., $\delta_2(\lambda) = 0$, and hence, equation (33) gives

   $$\operatorname{Sin}\delta = \frac{\operatorname{Cos}\epsilon \cdot \operatorname{Sin}\beta}{\mathcal{R}}.$$

   (c) When $\beta \neq 0°$ and $\delta_1(\lambda) = \epsilon$, in other words, when the celestial object has a non-zero-latitude and its first declination is equal to the ecliptic obliquity, the object lies on the solstitial colure. The solstitial colure is a great circle passing through the celestial poles and the solstitial points; shown in Figure 1 as the circle ○PR′RT′T. In this case, the second declination of the object is also equal to the ecliptic obliquity, i.e., $\delta_2(\lambda) = \epsilon$, and hence, equation (33) gives

   $$\operatorname{Sin}\delta = \operatorname{Sin}(\epsilon \pm \beta) \text{ or } \delta = \epsilon \pm \beta,$$

   where $\epsilon \pm \beta$ represents a special case of the curve of true declination with $\delta_2(\lambda) = \epsilon$.
2. Mullā Farīd repeats these three special cases in his *Zīj-i Shāh Jahānī* Discourse II.6, passages [5], [6], and [7]. These were then translated by Nityānanda in his *Siddhāntasindhu* Part II.6, $[5]_{\text{prose}}$, $[6]_{\text{prose}}$, and $[7]_{\text{prose}}$ (see Misra 2021: pp. 86, 88, 92, and 94). The *Sarvasiddhāntarāja* I.*spa·krā* excludes these prose passages in its recension of the second method (see Table 1).

Among Sanskrit authors, Mahendra Sūri, in his *Yantrarāja* I.46–48, and his student Malayendu Sūri's commentary on these verses, discuss a method to compute the true declination that closely resembles the second method described here (see Plofker 2000: 42–43) However, their expressions of equation (33) use the first declination instead of the second. This is geometrically inaccurate, and as Plofker speculates, their use of the first declination is either a 'convenient approximation or a very natural mistake' (p. 43).

4.6 THE THIRD METHOD OF TRUE DECLINATION

Nityānanda's describes the third method to compute the true declination of a celestial object in *Sarvasiddhāntarāja* I.*spa·krā*, verse 12, as follows:

> परस्फुटक्रान्तिभवज्यका गुणा सदृक्षबाहुज्यकया ऽधरीकृता ॥
> तदीय चापं भवति स्फुटापमो दिगस्य संयोगवियोगदिक्समा ॥ १२ ॥
>
> *parasphuṭakrāntibhavajyakā guṇā sadṛkṣabāhujyakayā 'dharīkṛtā* ||
> *tadīya cāpaṃ bhavati sphuṭāpamo digasya saṃyogaviyogadiksamā* || 12 ||
>
> The Sine of the maximum true declination (*para-sphuṭa-krānti-bhava-jyakā*), having been multiplied (*guṇā*) by the Sine of the congruent arc (*sadṛkṣa-bāhu-jyakā*) [and] having been lowered (*adharī-kṛtā*), its arc (*cāpa*) becomes the true declination (*sphuṭa-apama*). Its direction (*diś*) is the same (*sama*) as the direction of the conjunction or the disjunction (*saṃyoga-viyoga-diś*). 12

In other words,

$$\begin{matrix}\text{true}\\ \text{declination}\end{matrix} = \text{arcSin}\left[\frac{\text{Sin}\,(\text{maximum true declination})\cdot\text{Sin}\,\big(\text{congruent arc}\big)}{\textit{sinus totus}\ (\text{or Radius})}\right],$$

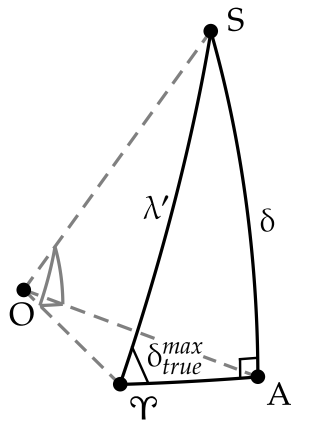
or expressed mathematically, $\delta = \text{arcSin}\left(\frac{\text{Sin}\,\delta_{true}^{max}\cdot\text{Sin}\,\lambda'}{\mathcal{R}}\right)$ where

Mathematical expression	*Sanskrit expression*	*English translation*
δ	*sphuṭa-apama*	true declination,
$\text{Sin}\,\delta_{true}^{max}$	*para-sphuṭa-krānti-bhava-jyakā*	Sine of the maximum true declination,
$\text{Sin}\,\lambda'$	*sadṛkṣa-bāhu-jyakā*	Sine of the congruent arc, and
$1/\mathcal{R}$	(*adharī-kṛtā*)	having been lowered, i.e., divided by the Radius or *sinus totus*.

4.6.1 Derivation of the third method

The mathematical expression of the third method of true declination can be derived as follows:

1. The excerpt (from Figure 1) below, to the left, shows, the right spherical triangle $\triangle \Upsilon AS$ with $\widehat{\Upsilon S}$ as the congruent arc λ', $\widehat{SA}$ as the true declination δ, $\sphericalangle SA\Upsilon = 90°$, and $\sphericalangle S\Upsilon A$ as the angle corresponding to the arc of maximum true declination δ^{max}_{true}. The arc $\widehat{\Upsilon S}$ is a part of the great circle congruent to the ecliptic (identified here, as the orbit of the celestial object) such that the latitude β and the declination δ are both similarly oriented (northwards); in other words, $\delta^{max}_{true} = \epsilon + \beta'_+$. Applying the spherical law of sines to triangle $\triangle \Upsilon AS$ gives



$$\frac{\sin \widehat{SA}}{\sin \sphericalangle S\Upsilon A} = \frac{\sin \widehat{S\Upsilon}}{\sin \sphericalangle SA\Upsilon}$$ or equivalently,

$$\sin \delta = \sin \lambda' \cdot \sin \delta^{max}_{true}. \qquad (34)$$

Written in terms of a non-unitary Radius (*sinus totus*), we find

$$\mathrm{Sin}\, \delta = \frac{\mathrm{Sin}\, \lambda' \cdot \mathrm{Sin}\, \delta^{max}_{true}}{\mathcal{R}}. \qquad (35)$$

Excerpt of $\triangle \Upsilon AS$ from Figure 1.

This allows us to compute the true declination δ as

$$\delta = \mathrm{arcSin}\left(\frac{\mathrm{Sin}\, \lambda' \cdot \mathrm{Sin}\, \delta^{max}_{true}}{\mathcal{R}}\right). \qquad (36)$$

The computations of the quantities $\mathrm{Sin}\, \lambda'$ and $\mathrm{Sin}\, \delta^{max}_{true}$ are discussed in § 4.3.1, equation (18), and § 4.3.2, equations (21) and (22) respectively. The direction of the true declination is the same as the direction of the arc of maximum true declination δ^{max}_{true}; see discussions in § 4.3.2.

Remark The compound *adharīkṛtā* 'having been lowered' in verse 12b refers to division by sixty, i.e., divided by the Radius or the *sinus totus*. Its use is comparable to the operation of lowering (*adharī-kṛ*) discussed in note 1 on p. 120, in the context of verse 9d. Once again, MS Bn.II parses the word *adharīkṛtā* in the middle of the verse on line 2 of f. 63v as *ṣaṣṭi-bhajanam-adh[y]arīkaraṇam-ucyate* 'division by sixty is called lowering'. Here, we find a clear parallel between the Persian expression *munḥaṭṭ kardan* 'to make low' and the Sanskrit *adhari-karaṇa*, 'making low'.

4.6.2 Historical testimonies of the third method

The third method of true declination, expressed mathematically in equation (36), can also be found in various Islamicate works (prior to Mullā Farīd's *Zīj-i Shāh Jahānī*). For example,

1. the second method to determine the distance of a star from the equator in al-Bīrūnī's *Kitāb Maqālīd ʿilm al-Hayʾa* (c. 994) (see Debarnot 1985: 214);[40]
2. the third method of determining the declination of a celestial body in Ibn Yūnis' *al-Zīj al-Kabīr al-Ḥākimī* (1003) (see King 1972: 39.1(c) on pp. 295–296);
3. the rule in chapters 1 and 2 of al-Kāshī's *Zīj al-Khāqānī* (c. 1413/1414), Treatise IV (see Kennedy 1985: 9) where the rule appear on f. 168r: 7 with its proof on f. 174v:6 of MS London India Office Persian 430 (Ethé 2232) of the *Zīj al-Khāqānī*; and
4. the third method in Sulṭān Ulugh Beg's *Zīj-i Jadīd-i Sulṭānī* Discourse II.5 (published 1438–1439) (see Sédillot 1853: 90–91), and repeated near-verbatim in Mullā Farīd's *Zīj-i Shāh Jahānī* Discourse II.6).[41]

Remark al-Bīrūnī also discusses a similar method of computing the distance of a star from the equator (with a slightly different derivation) in his *Kitāb Taḥdīd al-Amākin* (c. eleventh century), Chapter V.63: second method (see Kennedy 1973: 121), and also in his *al-Qānūn al-Masʿūdī* (c. 1030) (see Kennedy 1974: p. 65, with reference to *Chapter 4, On the extraction of the distance of a star having (non-zero) latitude from the celestial equator* on pp. 390–394 in Hyderabad-Dn., 1954–56 printed edition of al-Bīrūnī's *al-Qānūn al-Masʿūdī* (*Canon Masudicus*), three volumes.)

40 al-Bīrūnī's second method slightly differs from Nityānanda's rule in equation (36). Essentially, the arguments λ' and β' in equation (36) are expressed as complements $\overline{\lambda'}$ and $\overline{\beta'}$, and hence, the Sines in the numerator on the right-hand side of the equation appear as Cosines in al-Bīrūnī's expression.

41 Like al-Bīrūnī, Ulugh Beg's and Mullā Farīd's methods also use the expression $\overline{\lambda'}$. This quantity, understood as the distance of the celestial object from the "circle passing through the four poles" (i.e., from the solstitial colure), is discussed at the end of the § 4.2.4. Also, Ulugh Beg and Mullā Farīd both refer to the arcs of maximum latitude β' and maximum true declination δ^{max}_{true} as the first and second arcs respectively (see p. 114.)

Among Sanskrit authors, Mahendra Sūri's verses, in his *Yantrarāja* I. 43–44, and Malayendu Sūri's commentary on these verses, discuss a method similar Nityānanda's third method of true declination (see Plofker 2000: 42–43). As noted before (in remark 2 on p. 121), Mahendra Sūri considers $\frac{\operatorname{Sin}\beta}{\operatorname{Sin}\lambda'}$ as the maximum latitude β' (instead of $\frac{\operatorname{Sin}\beta'}{\mathcal{R}}$, i.e., $\sin\beta'$, following equation (20)). With this value, Mahendra Sūri calculates the maximum true declination δ_{true}^{max} as $\epsilon \pm \beta'$ (from equation (22)), and subsequently

$$\operatorname{Sin}\Delta\delta = \frac{\operatorname{Sin}\lambda' \cdot \operatorname{Sin}\delta_{true}^{max}}{\mathcal{R}} \quad \text{(see Plofker 2000: 42).} \tag{37}$$

The right-hand side of equation (37) is identical to what is seen in equation (35); however, Mahendra Sūri takes this result to equal the Sine of a 'declination correction' $\Delta\delta$ instead of Sine of the true declination δ. The true declination is this declination correction, calculated as the inverse Sine of equation (37), 'increased or diminished by the latitude when [the longitude of] the star and the latitude are in the same or different hemispheres'; in other words, $\delta = \beta \pm \Delta\delta$ in a direction 'either south or north with respect to the equator' (Plofker 2000: *Yantrarāja*: I.44cd–45acd, p. 41). As Plofker suggests, Mahendra Sūri's method appears to be a possible corruption or confusion of the Ibn Yūnis's method (pp. 42–43).

4.6.3 Special case of celestial objects stationed at the ecliptic pole

In the *Sarvasiddhāntarāja* I.*spa·krā*, verse 13, Nityānanda discusses the special case when the celestial object—specifically identified as a star (*uḍu*)— is stationed at the ecliptic pole. For such a star,

$$\operatorname{Sin}\begin{pmatrix}\text{true}\\ \text{declination}\end{pmatrix} = \operatorname{Sin}\begin{pmatrix}\text{complement of}\\ \text{greatest declination}\end{pmatrix} = \operatorname{Cos}\begin{pmatrix}\text{first declination}\\ \text{of the longitude}\end{pmatrix}.$$

or expressed mathematically, $\operatorname{Sin}\delta = \operatorname{Sin}(90° - \epsilon) = \operatorname{Cos}\delta_1(\lambda)$ where

Mathematical expression	*Sanskrit expression*	*English translation*
$\operatorname{Sin}(\delta)$	*spaṣṭa-krānti*	Sine of the true declination,
ϵ	*parama-krānti*	greatest declination, i.e., obliquity of the ecliptic, and
$\operatorname{Cos}[\delta_1(\lambda)]$	*dyujīvā*	day-Sine, i.e., Cosine of the first declination of the longitude, see § 4.2.2.

Derivation Figure 8 shows a star S stationed at the north ecliptic pole P′ (with longitude $\lambda = 270°$) in the celestial sphere. The circle ○NϒHΩ represents the great circle congruent to the ecliptic and passing through the star, with point H being coincident to the north ecliptic pole P′ (and also the position S of the star in its orbit). In this configuration,

1. the latitude of the star β (i.e., arcs $\widehat{P'D}$, $\widehat{P'Q}$, or $\widehat{P'R'}$) is always 90° for any value of its longitude λ;
2. the first declination $\delta_1(\lambda)$ (i.e., arc $\widehat{QA}$ that measures the distance between the ecliptic and the equator along the great circle passing through the celestial pole and the ecliptical projection point, indicated in Figure 8 as Q) corresponds to the ecliptic obliquity ϵ (i.e., arc $\widehat{T'T}$) when $\lambda = 270°$, i.e., $\delta_1(\lambda) = \epsilon$;
3. the congruent arc (*sadṛkṣa-bāhu*) $\widehat{\Upsilon P'}$ or λ' is also 90°;
4. the maximum latitude β' (i.e., arcs $\widehat{P'T'}$ or $\widehat{HT'}$) is simply, $\sin\beta' = \dfrac{\sin\beta}{\sin\lambda'} = 1$ using equation (19), implying β' is 90°; and
5. the maximum true declination δ_{true}^{max} (i.e., arc $\widehat{HT}$) is $180° - (\epsilon + \beta')$ because $\beta' + \epsilon > 90°$, see equation (23). This implies that
$$\delta_{true}^{max} = 180° - (\epsilon + 90°) = 90° - \epsilon.$$

Therefore, using equation (34), the true declination of the star positioned at the north ecliptic pole (i.e., arcs $\widehat{SA}$ or $\widehat{HT}$) is

$$\sin\delta = \sin\lambda' \cdot \sin\delta_{true}^{max} = \sin 90° \cdot \sin(90° - \epsilon) = \sin(90° - \epsilon).$$

In other words, $\sin\delta = \sin(90° - \epsilon) = \cos\epsilon = \cos\delta_1(\lambda) = \sin\left[90° - \delta_1(\lambda)\right]$, or

$$\delta = 90° - \epsilon = 90° - \delta_1(\lambda). \tag{38}$$

Equation (38) is the mathematical statement of what Nityānanda describes in verse 13: the true declination is directly obtained from the maximum declination (or ecliptic obliquity) for any star stationed at the ecliptic pole.

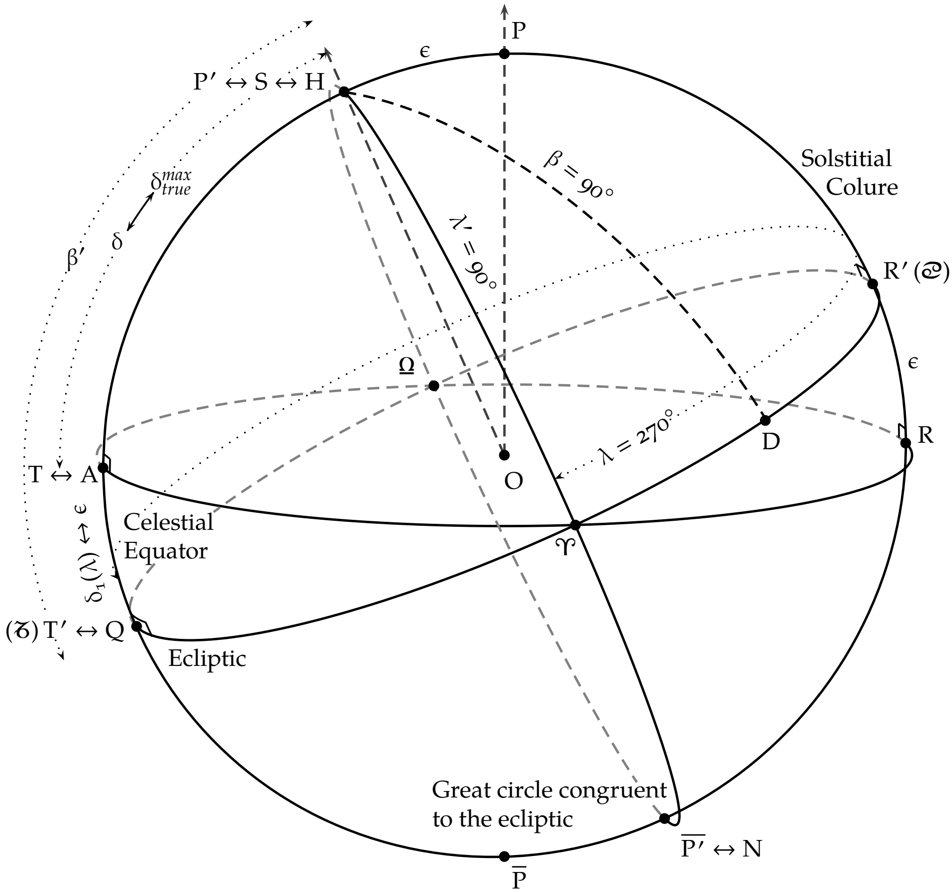


Figure 8: The celestial sphere with a star S stationed at the north ecliptic pole P′ (shown here with longitude $\lambda = 270°$). The circle ○N♈HΩ represents the great circle congruent to the ecliptic such that orbital point H, the position of the star at S, and the ecliptic pole P′ are coincident. The true declination δ of the star can then be seen to be equal to the complement of the ecliptic obliquity ϵ, or equivalently, the complement of the first declination of the star.

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INDEX OF MANUSCRIPTS

(*References are to page numbers in this article.*)

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APPENDICES

A ON COMPUTING THE TRUE DECLINATION IN MEDIEVAL SANSKRIT TEXTS

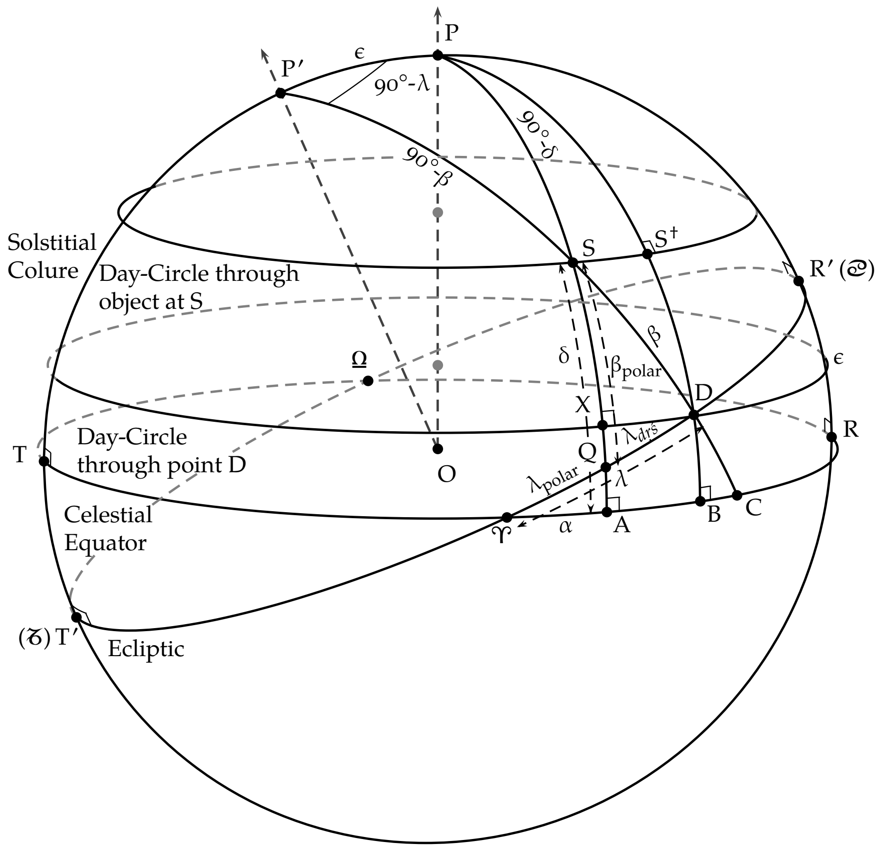

Figure A1: Celestial sphere showing the position S of a celestial object with equatorial coordinates $(\overset{\frown}{SA}, \overset{\frown}{\Upsilon A})$ or (δ, α); ecliptic coordinates $(\overset{\frown}{SD}, \overset{\frown}{\Upsilon D})$ or (β, λ); and polar coordinates $(\overset{\frown}{SQ}, \overset{\frown}{\Upsilon Q})$ or $(\beta_{polar}, \lambda_{polar})$.

Figure A1 shows a celestial object (star or planet) positioned at S in the celestial sphere with arcs $\overset{\frown}{T\Upsilon R}$ and $\overset{\frown}{T'\Upsilon R'}$ being the celestial equator and ecliptic respectively. The coordinates of this celestial object are $(\overset{\frown}{SA}, \overset{\frown}{\Upsilon A})$ or (δ, α) in equatorial coordinates, $(\overset{\frown}{SD}, \overset{\frown}{\Upsilon D})$ or (β, λ) in ecliptic coordinates, and $(\overset{\frown}{SQ}, \overset{\frown}{\Upsilon Q})$ or $(\beta_{polar}, \lambda_{polar})$ in polar coordinates.

The arcs $\widehat{XD}$ and $\widehat{SS^{\dagger}}$ in Figure A1 are parts of the small day-circles (*dyu-vṛtta*) of the celestial object passing through points D and S respectively. Point D is the intersection of the secondary to the ecliptic passing through the celestial object at S and the ecliptic. In Sanskrit astronomy, point S is considered as the position of the disk (*bimba*) of the planet (*graha*), while the ecliptic point D represents its image (*pratibimba*) or projection (*vikṣepa*). The day-circle through D intersects the declination arc $\widehat{SA}$ at point X and the day-circle through point S intersects the polar arc through D, i.e., $\widehat{PD}$, at point $S^{\dagger}$. The equatorial points B and C represent the points of intersections of the polar arc $\widehat{PD}$ and the ecliptic arc $\widehat{P'D}$ with the celestial equator respectively.

A.1 ON THE USE OF POLAR COORDINATES

The arc of true declination $\widehat{SA}$ of a celestial object can be decomposed into polar coordinates as

$$\widehat{SA} = \widehat{QA} \pm \widehat{SQ}, \text{ or equivalently } \delta = \delta'(\lambda_{\text{polar}}) \pm \beta_{\text{polar}}, \quad \text{(A1)}$$ [1]

where $\widehat{QA} = \delta'(\lambda_{\text{polar}})$ is the declination of ecliptic point Q that lies at an ecliptic longitude of λ_{polar} from the vernal equinoctial point ♈ (0° Aries). Historically, Sanskrit astronomers often appropriated the polar coordinates (β_{polar}, λ_{polar}) of a celestial object.

The polar coordinate system is a non-orthogonal coordinate system. Here, the polar latitude β_{polar} is measured with respect to the equatorial pole P, whereas the polar longitude λ_{polar} is measured with respect to the ecliptic pole P′. Hence

- the polar longitude λ_{polar} (arc $\widehat{♈Q}$) can be regarded as the ecliptic longitude λ (arc $\widehat{♈D}$) corrected by the ecliptic arc $\lambda_{dṛś}$ (arc $\widehat{QD}$). In Sanskrit astronomy, this ecliptic arc is called the *āyana-dṛkkarma* 'visibility correction due to ecliptic deviation'.
- And the polar latitude β_{polar} (arc $\widehat{SQ}$) can be thought of as a transformation of the ecliptic latitude β (arc $\widehat{SD}$) along the direction of the declination δ (arc $\widehat{SA}$); in other words, $\beta \rightsquigarrow \beta_{\text{polar}}$ along the direction of δ.

The Indian and Greek methods of converting the ecliptic longitude of a heavenly body into its polar longitude is discussed in Sengupta (1931: 20–24). Also,

1 The latitude is added to or subtracted from the declination depending on the orientation of the celestial object. Compare discussions on the curve of true declination (*sphuṭa-apama-aṅka*) in § 4.2.1.

Plofker (2000: 40) discusses a typical Sanskrit method of (approximately) calculating the polar coordinates from which, in theory, the true declination of the celestial object could be determined. However, most *siddhāntic* texts prescribe a different (approximate) method for this calculation.

A.2 ON THE USE OF ECLIPTIC COORDINATES

Alternatively, the true declination of the celestial object δ, the equivalent arcs $\overparen{SA}$ and $\overparen{S^{\dagger}B}$ in equatorial coordinates, can also be decomposed as $\overparen{SA} = \overparen{XA} \pm \overparen{SX}$ and $\overparen{S^{\dagger}B} = \overparen{DB} \pm \overparen{S^{\dagger}D}$.[2] This implies,

$$\delta = \overparen{SA} = \overparen{S^{\dagger}B} = \delta'(\lambda) \pm \overparen{SX} \text{ or } \delta'(\lambda) \pm \overparen{S^{\dagger}D}, \tag{A2}$$

where the $\overparen{XA} = \overparen{DB} = \delta'(\lambda)$ is the declination of the ecliptic projection point D. The value of $\overparen{DB}$ is computed using the spherical law of sines applied to right spherical triangle $\triangle\Upsilon BD$ and $\triangle\Upsilon RR'$ such that

$$\operatorname{Sin}\overparen{DB} = \operatorname{Sin}\delta'(\lambda) = \frac{\operatorname{Sin}\epsilon \cdot \operatorname{Sin}\lambda}{\mathcal{R}},$$

a method commonly referred to as the method of declination. (This is also the expression of the first declination, see Appendix B.)

In most medieval Sanskrit *siddhānta*s, the arc $\overparen{SX}$ (or $\overparen{S^{\dagger}D}$) was often considered equal to the ecliptic latitude $\overparen{SD}$ or β on account of the smallness of a planet's deviation from the ecliptic (i.e., β being small, $\overparen{SX} = \overparen{S^{\dagger}D} \approx \beta$).[3] This allowed the true declination δ in equation (A2) to be approximated as

$$\delta = \overparen{SA} = \overparen{S^{\dagger}B} \approx \delta'(\lambda) \pm \beta. \tag{A3}$$

2 See footnote 1 on p. 146

3 The ecliptic latitude β of a planet is typically calculated (or approximated) using a method similar to the method of declinations. For instance, the Moon's orbit is at $\iota^{\circ} \sim 4;30^{\circ}$ to the ecliptic with λ_{Ω} being the longitude of its ascending node. Hence, its ecliptic latitude $\beta_{☾}$ corresponding to its ecliptic longitude $\lambda_{☾}$ can be expressed as

$$\operatorname{Sin}\beta_{☾} = \frac{\operatorname{Sin}\left(\lambda_{☾} - \lambda_{\Omega}\right) \cdot \operatorname{Sin}\iota^{\circ}}{\mathcal{R}}.$$

In its approximate form,

$$\beta_{☾} \approx \frac{\operatorname{Sin}\left(\lambda_{☾} - \lambda_{\Omega}\right) \cdot 4;30^{\circ}}{\mathcal{R}}$$

as $\operatorname{Sin}\beta_{☾} \sim \beta_{☾} \;\forall\; \beta_{☾} \in [0^{\circ}, 4;30^{\circ}]$ and $\operatorname{Sin} 4;30^{\circ} \sim 4;30^{\circ}$.

REMARKS

1. According to Sudhākara Dvivedi's Sanskrit commentary, the *Nūtana-tilaka* (1902), on the *Brāhmasphuṭasiddhānta* X.15–16 (628), Brahmagupta describes the true declination (*sphuṭa-krānti*) as the sum or difference of its ecliptic declination (*dhruva-krānti*) $\widehat{\mathrm{DB}}$ and its true (polar) latitude (*spaṣṭa-śara*) $\widehat{\mathrm{SX}}$. The true latitude being small, it is then simply approximated by the ecliptic latitude $\widehat{\mathrm{SD}}$ following equation (A3) (see Sharma 1966*a*: 585–589).
   The same approximation is also repeated in the calculation of the true declination of the Moon in the *Brāhmasphuṭasiddhānta* VII.5. As Sharma (1966*b*: 483–485) describes (in the *upapatti* on p. 484), the arc $\widehat{\mathrm{SD}}$ is understood as the mean latitude (*madhyama-śara*) while the arc $\widehat{\mathrm{DB}}$ is the mean declination (*madhyama-krānti*). In contrast, the arc $\widehat{\mathrm{SX}}$ (or $\widehat{\mathrm{S^{\dagger}D}}$) is considered to be the true latitude (*spaṣṭa-śara*). With the true latitude being small (when compared to the mean latitude), the true declination is again approximated following equation (A3).
   Brahmagupta's *Khaṇḍakhādyaka* I.3.7cd (665) also states equation (A3) as the general rule to compute the true declination of a planet (see Chatterjee 1970: p. 96 in Volume I and p. 59 in Volume II).
2. Bhāskara I, in his *Mahābhāskarīya* VI.8 (c. seventh century) (see Shukla 1960: p. 38 and pp. 188–189) and the (modern) *Sūryasiddhānta* II.58 (c. 800) (see Bhaṭṭācārya 1891: 82) both describe equation (A3) as the method to calculate the true declination of a planet (Moon).
3. Lalla, in his *Śiṣyadhīvṛddhidatantra* I.9.1–2 (c. late eight or early ninth century) also uses the approximate method of equation (A3) to compute the true declination (*sphuṭa-krānti*) of the Moon from its mean declination (*madhyama-krānti*) and its latitude (*vikṣepa*) (see Chatterjee 1981: p. 132 in Part I and pp. 162–163 in Part II).
4. Vaṭeśvara, in his *Vaṭeśvarasiddhānta* I.6.21ab (904) (see Shukla 1986: p. 275 in Part I and pp. 542–543 in Part II) and Āryabhaṭa II, in his *Mahāsiddhānta* I.3.38b (c. late ninth century) (see S. Dvivedi 1910: 65) reiterate the rule in equation (A3) to compute the true declination of a planet (Moon).
5. Śrīpati, in his *Siddhāntaśekhara* XXI.7 (c. eleventh century) describes a method to compute the true declination of the Moon identical to equation (A3) (see Miśra 1932: p. 439 in Part I).

To calculate the arc $\widehat{\mathrm{SX}}$ (or arc $\widehat{\mathrm{S^{\dagger}D}}$) precisely, instead of approximating it with the ecliptic latitude $\widehat{\mathrm{SD}}$, the right-angled convex triangle $\triangle$SXD (or $\triangle\mathrm{DS^{\dagger}S}$)

is approximated as a right-angled planar triangle. This then gives

$$\begin{aligned} \widehat{\mathrm{SX}} \sim \mathrm{SX} = \frac{\mathrm{SD} \cdot \mathrm{Cos}\, \angle \mathrm{XSD}}{\mathcal{R}} = \frac{\beta \cdot \mathrm{Cos}\, \angle \mathrm{XSD}}{\mathcal{R}} = \frac{\beta \cdot \sqrt{\mathcal{R}^2 - \mathrm{Sin}\, \angle \mathrm{XSD}}}{\mathcal{R}} \text{ or} \\ \widehat{\mathrm{S^{\dagger}D}} \sim \mathrm{S^{\dagger}D} = \frac{\mathrm{SD} \cdot \mathrm{Cos}\, \angle \mathrm{SDS^{\dagger}}}{\mathcal{R}} = \frac{\beta \cdot \mathrm{Cos}\, \angle \mathrm{SDS^{\dagger}}}{\mathcal{R}} = \frac{\beta \cdot \sqrt{\mathcal{R}^2 - \mathrm{Sin}\, \angle \mathrm{SS^{\dagger}D}}}{\mathcal{R}}. \end{aligned} \tag{A4}$$

With parallel great-circle arcs $\widehat{\mathrm{PA}}$ and $\widehat{\mathrm{PB}}$ intersected by the great-circle arc $\widehat{\mathrm{P'C}}$ in Figure A1, we can consider the (planar-approximate) $\angle\mathrm{XSD}$ or $\angle\mathrm{SS^{\dagger}D}$ equal in measure to the spherical angle $\sphericalangle\mathrm{PSP'}$. Thus, for the spherical triangle $\triangle\mathrm{PSP'}$, the spherical law of sines gives

$$\sin \sphericalangle\mathrm{PSP'} = \frac{\sin(90^\circ - \lambda) \cdot \sin\epsilon}{\sin(90^\circ - \delta)} = \frac{\sin(90^\circ + \lambda) \cdot \sin\epsilon}{\cos\delta}$$

since $\sin(90^\circ + x) = \cos(x) = \sin(90^\circ - x)$. Hence,

$$\begin{aligned} \mathrm{Sin} \sphericalangle\mathrm{PSP'} &= \frac{\mathrm{Sin}(90^\circ \pm \lambda) \cdot \mathrm{Sin}\,\epsilon}{\mathrm{Cos}\,\delta} \text{ (for non-unitary Radius or } \textit{sinus totus}) \\ \Rightarrow \mathrm{Sin} \sphericalangle\mathrm{PSP'} &= \frac{\mathcal{R} \cdot \mathrm{Sin}\,\delta'(90^\circ \pm \lambda)}{\mathrm{Cos}\,\delta} \quad \because \mathrm{Sin}\,\delta'(x^\circ) = \frac{\mathrm{Sin}\,x^\circ \cdot \mathrm{Sin}\,\epsilon}{\mathcal{R}}. \end{aligned} \tag{A5}$$

In Sanskrit astronomy, the quantity $\sphericalangle\mathrm{PSP'}$ is called the *āyana-valana* 'deflection due to ecliptic obliquity'. Its value is between 0° (when $\lambda = \pm 90^\circ$ and assuming $\beta = 0^\circ$) and ϵ (when $\lambda = 0^\circ$). Correspondingly, the denominator in equation (A5) can be approximated as

> $\mathrm{Cos}(\delta = \epsilon) \sim \mathcal{R} \because \epsilon \in [23.5^\circ, 24^\circ]$ when $\lambda = \pm 90^\circ$ and assuming $\beta = 0^\circ$ and $\mathrm{Cos}(\delta = 0^\circ) = \mathcal{R}$ when $\lambda = 0^\circ$.

This allows equation (A5) to be written as

$$\mathrm{Sin} \sphericalangle\mathrm{PSP'} \approx \mathrm{Sin}\,\delta'(90^\circ \pm \lambda) \quad \text{or} \quad \mathrm{Cos} \sphericalangle\mathrm{PSP'} \approx \mathrm{Cos}\,\delta'(90^\circ \pm \lambda). \tag{A6}$$

From equations (A4) and (A6), we find

$$\widehat{\mathrm{SX}} \sim \mathrm{SX} \text{ or } \widehat{\mathrm{S^{\dagger}D}} \sim \mathrm{S^{\dagger}D} = \frac{\beta \cdot \sqrt{\mathcal{R}^2 - \mathrm{Sin} \sphericalangle\mathrm{PSP'}}}{\mathcal{R}} \approx \frac{\beta \cdot \mathrm{Cos}\,\delta'(90^\circ \pm \lambda)}{\mathcal{R}}, \tag{A7}$$

and hence, the true declination arc $\widehat{\mathrm{SA}}$ or $\widehat{\mathrm{S^{\dagger}B}}$ in equation (A2) can then be approximated as

$$\begin{aligned} \widehat{\mathrm{SA}} = \widehat{\mathrm{S^{\dagger}B}} = \delta \approx \delta'(\lambda) \pm \frac{\beta \cdot \sqrt{\mathcal{R}^2 - \mathrm{Sin} \sphericalangle\mathrm{PSP'}}}{\mathcal{R}} \text{ or} \\ \widehat{\mathrm{SA}} = \widehat{\mathrm{S^{\dagger}B}} = \delta \approx \delta'(\lambda) \pm \frac{\beta \cdot \mathrm{Cos}\,\delta'(90^\circ \pm \lambda)}{\mathcal{R}}. \end{aligned} \tag{A8}$$

REMARKS

1. Bhāskara II, in his *Siddhāntaśiromaṇi* I.8.3 (1150) describes two methods to compute the true declination of a planet by applying a correction to the declination (*krānti-saṃskāra*); in other words, adding a correction to the arc $\widehat{DB}$ or $\delta'(\lambda)$. Bhāskara II's methods are identical to the two approximations of the true declination arc $\widehat{SA} = \widehat{S^{\dagger}B}$ in equation (A8) (see Chaturvedi 1981: 276–278).
   Bhāskara II repeats the second rule of equation (A8) (using Cosine) in his *Siddhāntaśiromaṇi* II.9.10 to calculate the rectified latitude (*spaṣṭa-śara*). The rectified latitude is then added to the mean declination (*madhyama-krānti*) in equation (A8) (see Chaturvedi 1981: 433).
2. The rules for computing the true declination of the Sun and the Moon according to the Nila School of Kerala (South India),[4] especially the methods of Nīlakaṇṭha Somayājī (c. 1444–c. 1545) in his *Tantrasaṅgraha* (1501) and Śaṅkara Vāriyar (c. 1500–c. 1560) in his prose commentary *Laghuvivṛti* on the *Tantrasaṅgraha*, are discussed in Plofker (2002: 87–91) and Ramasubramanian and Sriram (2011: 359–369). Also, see Hirose (2017: 234–238) where the method of determining the corrected latitude and true declination in Parameśvara's *Goladīpikā* (c. *post* 1432–c. *ante* 1443) is discussed.

4 The history of the Nila (Niḷā) school of mathematicians, beginning with Mādhava of Saṅgamagrāma (c. 1340–c. 1425), is described Divakaran (2018: *Chapter 9: The Nila Phenomenon* on pp. 257–290).

B ON DERIVING THE SECOND DECLINATION FROM THE FIRST IN NITYĀNANDA'S *SARVASIDDHĀNTARĀJA* I.4.49–50AB

In the topic on three questions (*tripraśna*) [on direction, place, and time] in the chapter on computations (*gaṇitādhyāya*) of the *Sarvasiddhāntarāja*, Nityānanda derives the second declination from the first. His begins his discussion on the two types of declination with the statement[5]

> अथ क्रान्तिसूत्रगतक्रान्तेः शरसूत्रगतक्रान्त्यानयनम् ।
>
> *atha krāntisūtragatakrānteḥ śarasūtragatakrānty ānayanam* |
>
> Now, [the rule for] calculating (*ānayana*) the [second] declination (*krānti*) associated with the line-of-latitude (*śara-sūtra*) from the [first] declination (*krānti*) associated with the line of declination (*krānti-sūtra*).

In Figure B1, the arc $\widehat{\mathrm{DB}}$ is the arc of first declination and the arc $\widehat{\mathrm{DC}}$ is the arc of the second declination, both measured from point D, the intersection of the secondary to the ecliptic passing through the celestial object at S and the ecliptic arc $\widehat{\mathrm{T'\Upsilon R'}}$.

1. The polar longitudinal arc $\widehat{\mathrm{PDB}}$ passing through the ecliptical projection of the celestial object, i.e., point D, produces the arc of first declination $\widehat{\mathrm{DB}}$. Hence, the measure of the first declination is associated with the line of declination (*krānti-sūtra*).
2. The secondary to the ecliptic, i.e., arc $\widehat{\mathrm{P'SDC}}$ is associated with the line of latitude (*śara-sūtra*) where arc $\widehat{\mathrm{SD}}$ is the latitude (*śara*) of the celestial object. Hence, the second declination arc $\widehat{\mathrm{DC}}$ is considered in connection to the latitudinal direction.

In verses 49–50ab of *tripraśnādhikāra*, Nityānanda describes a method to compute the second declination (*dvitīyā-krānti*) from the first declination (*krānti*):

> या कोटितो भवेत्क्रान्तिः सा विलोमानि गद्यते ॥
> विलोमक्रान्तिकोटिज्योद्धृता त्रिज्या गुणा पुनः ॥ ४९ ॥
> क्रान्तिज्या फलजं चापं द्वितीया क्रान्तिरुच्यते ॥ ५०[प्र,द्वि]
>
> *yā koṭito bhavet krāntiḥ sā vilomāni gadyate* ||
> *vilomakrāntikoṭijyoddhṛtā trijyā guṇā punaḥ* || 49 ||

[5] The numbering of the verses in the *tripraśnādhikāra* of the *Sarvasiddhāntarāja* is different in different manuscripts. I follow MS Np: National Archives Nepal, NAK 5.7255 (NGMCP Microfilm Reel № B 354/15) as it appears to be the most uniform in its verse-numbering.

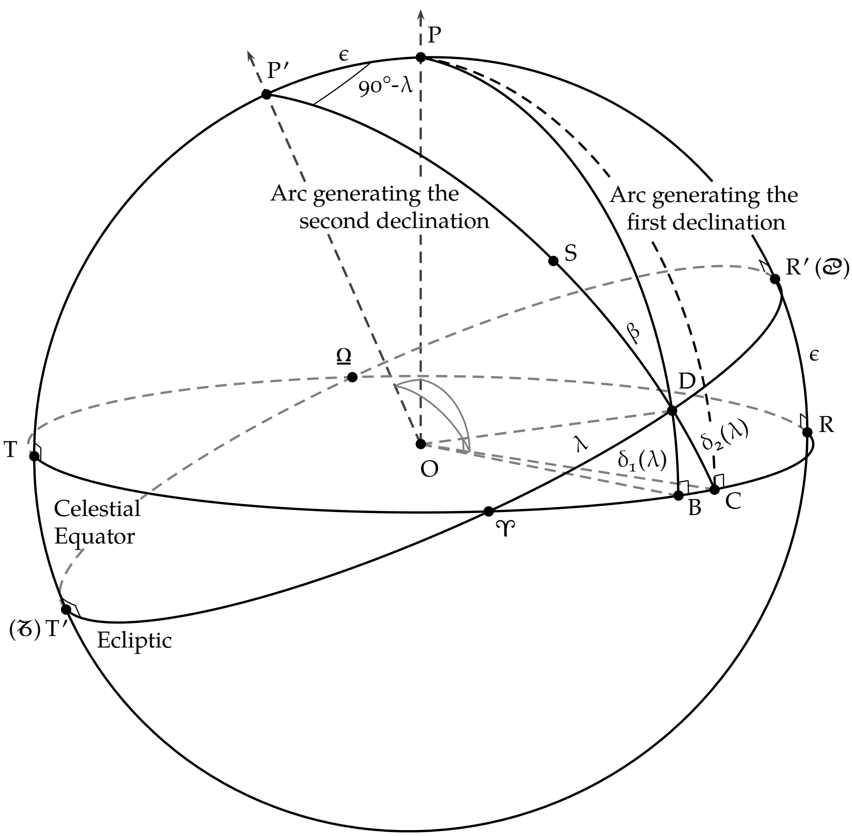

Figure B1: The celestial sphere showing the spherical triangles inscribed by the celestial equator, the ecliptic, and their different secondary circles with respect to a celestial object positioned at S.

krāntijyā phalajaṃ cāpaṃ dvitīyā krāntir ucyate || 50ab

Whatever [ecliptic] declination (*krānti*) is [derived] from the complement [of the longitude] (i.e., from the *koṭi*), that is called the reverse (*viloma*) [declination]. The *sinus totus* (*trijyā*) divided by the Cosine of the reverse declination (*viloma-krānti-koṭijyā*) and again multiplied 49...

...by the Sine of the [ecliptic] declination (*krānti-jyā*). The arc (*cāpa*) produced from the result (*phala*) is called the second declination (*dvitīyā-krānti*). 50ab

To understand this method, we first apply the spherical law of sines to the right spherical triangle $\triangle$DBC in Figure B1:

$$\frac{\sin \widehat{DC}}{\sphericalangle DBC} = \frac{\sin \widehat{DB}}{\sphericalangle DCB} \Rightarrow \sin \widehat{DC} = \frac{\sin \widehat{DB} \cdot \sin \sphericalangle DBC}{\sin \sphericalangle DCB}, \text{ or}$$
$$\sin \delta_2(\lambda) = \frac{\sin \delta_1(\lambda)}{\sin \sphericalangle DCB} \quad \because \sin \sphericalangle DBC = \sin 90^\circ = 1. \tag{B1}$$

With $\sphericalangle DCB = \sphericalangle PCB - \sphericalangle PCP' = 90^\circ - \sphericalangle PCP'$, we can again apply the spherical law of sines to the spherical triangle $\triangle PCP'$ as

$$\frac{\sin \sphericalangle PCP'}{\sin \widehat{PP'}} = \frac{\sin \sphericalangle CP'P}{\sin \widehat{CP}} \Rightarrow \frac{\sin \sphericalangle PCP'}{\sin \epsilon} = \frac{\sin (90^\circ - \lambda)}{\sin 90^\circ}.$$

This gives $\sin \sphericalangle PCP' = \sin(\epsilon) \cdot \sin(90 - \lambda) = \sin \delta_1(90^\circ - \lambda)$, since $\sin \delta_1(x^\circ) = \sin \epsilon \cdot \sin x^\circ$. Thus, $\sin \sphericalangle PCP' = \sin \delta_1(90^\circ - \lambda)$ or $\sphericalangle PCP' = \delta_1(90^\circ - \lambda)$, and hence

$$\sphericalangle DCB = 90^\circ - \sphericalangle PCP' = 90^\circ - \delta_1(90^\circ - \lambda). \tag{B2}$$

From equations (B1) and (B2), it follows that

$$\sin \delta_2(\lambda) = \frac{\sin \delta_1(\lambda)}{\sin \left[90^\circ - \delta_1(90^\circ - \lambda)\right]} = \frac{\sin \delta_1(\lambda)}{\cos \delta_1(90^\circ - \lambda)}. \tag{B3}$$

For a non-unitary Radius (*sinus totus*), we have

$$\mathrm{Sin}\, \delta_2(\lambda) = \frac{\mathrm{Sin}\, \delta_1(\lambda) \cdot \mathcal{R}}{\mathrm{Cos}\, \delta_1(90^\circ - \lambda)}, \tag{B4}$$

which is the mathematical expression of the statement

$$\text{Sin (second declination)} = \frac{\text{Sin (first declination)} \cdot \textit{sinus totus} \text{ (or Radius)}}{\text{Cos}\begin{pmatrix}\text{first declination of ecliptic} \\ \text{longitude decreased from } 90^\circ\end{pmatrix}}$$

in verses 49–50a. Nityānanda refers to the first declination derived from the complement (*koṭi*) of the longitude, i.e., $\delta_1(90^\circ - \lambda)$, as the 'reverse declination' (*viloma-krānti*). Compare this with the Islamicate term 'inverse declination' (*al-māyl al-maʿkūs*) described in footnote 34 on p. 127. The arc Sine of the result on the right-hand side of equation (B4), when expressed in degrees etc., is the expression for the second declination in verse 50b above.

REMARKS

1. Munīśvara, in his auto-commentary on his *Siddhāntasārvabhauma* I.4.43–45 (1646), called the *Āśayaprakāśinī* or *Siddhāntatattvārtha*, also describes the declination as *krāntivṛttaviṣuvavṛttapradeśayor madhyasthāṃśāḥ* 'the degrees [of arc] situated in the middle of the region of the ecliptic (*krānti-vṛtta*) and the celestial equator (*viṣuvat-vṛtta*)', with the conditions that
   - when the arc is measured on the *dhruvaprotavṛtta* '[great] circle fixed to celestial pole (*dhruva*)' it is the first declination (simply called *krānti* 'declination') and
   - when the arc is measured on the *kadambaprotavṛtta* '[great] circle fixed to ecliptic pole (*kadamba*)' it is the second declination (called *anya-krānti* 'other declination') (see Ojhā 1978: 421).
2. It is interesting to note that Munīśvara's and Kamalākara's methods to compute the second declination (from the first declination) are also identical to Nityānanda's method; see equations (D1) and (E1). Both their methods rely on the identification of $\delta_1(90° - \lambda) \leftrightarrow \delta_1(90° + \lambda)$ which is discussed in equation (C2).

C ON THE EQUIVALENCE OF THE FIRST AND SECOND METHODS OF DECLINATION IN NITYĀNANDA'S *SARVASIDDHĀNTARĀJA* I.*SPA·KRĀ*.2–3

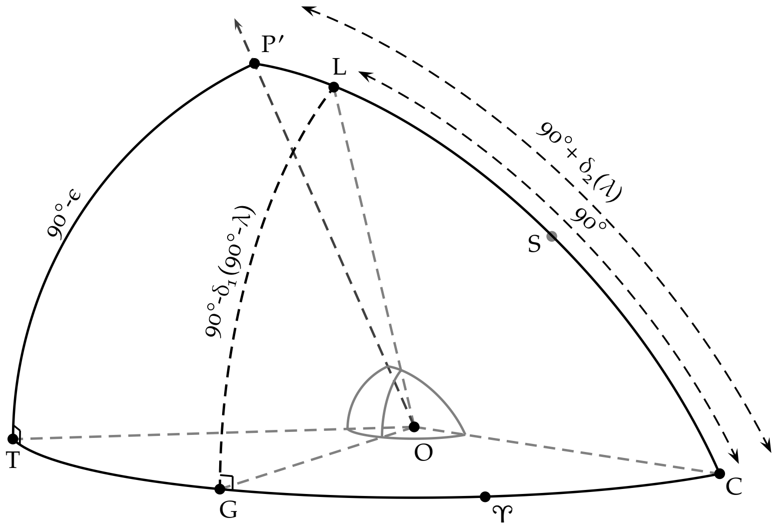

Figure C1: Right spherical triangles △P′TC and △LGC.

In verses 2 and 3, Nityānanda gives the following two formulae (equations (30) and (33) in §§ 4.4 and 4.5 respectively) to compute the Sine of the true declination (*sphuṭa-apama* or *sphuṭa-krānti*) δ:

$$\operatorname{Sin}\delta = \frac{\operatorname{Sin}\left[\delta_2(\lambda) \pm \beta\right] \cdot \operatorname{Cos}\delta_1(90^\circ + \lambda)}{\mathcal{R}} \text{ and } \operatorname{Sin}\delta = \frac{\operatorname{Cos}\epsilon \cdot \operatorname{Sin}\left[\delta_2(\lambda) \pm \beta\right]}{\operatorname{Cos}\delta_2(\lambda)}.$$

To see the equivalence of these formulae, we observe

$$\begin{aligned} \frac{\operatorname{Sin}\left[\delta_2(\lambda) \pm \beta\right] \cdot \operatorname{Cos}\delta_1(90^\circ + \lambda)}{\mathcal{R}} &= \frac{\operatorname{Cos}\epsilon \cdot \operatorname{Sin}\left[\delta_2(\lambda) \pm \beta\right]}{\operatorname{Cos}\delta_2(\lambda)} \\ \Rightarrow \frac{\operatorname{Cos}\delta_1(90^\circ + \lambda)}{\mathcal{R}} &= \frac{\operatorname{Cos}\epsilon}{\operatorname{Cos}\delta_2(\lambda)} \text{ or } \cos\delta_1(90^\circ + \lambda) = \frac{\cos\epsilon}{\cos\delta_2(\lambda)}. \end{aligned} \tag{C1}$$

This equality can be verified by looking at the right spherical triangles △P′TC and △LGC in Figure 1. These triangles are excerpted and redrawn in Figure C1. The triangles △P′TC and △LGC are right spherical triangles with the right angles ∢LGC and ∢P′TC, and sides:

1. $\widehat{\mathrm{P'T}} = \widehat{\mathrm{PT}} - \widehat{\mathrm{P'P}} = 90^\circ - \epsilon$,
2. $\widehat{\mathrm{LC}} = 90^\circ$, and
3. $\widehat{\mathrm{P'C}} = \widehat{\mathrm{LC}} + \widehat{\mathrm{LP'}} = 90^\circ + \underline{\delta_2(\lambda)} \quad \because \widehat{\mathrm{LP'}} = \widehat{\mathrm{DC}} = \delta_2(\lambda)$ from equation (3), and
4. $\widehat{\mathrm{LG}} = 90^\circ - \delta_1(90^\circ - \lambda) = \delta_1(\overline{\lambda})$.[6]

With the Rule of Four Quantities applied to right spherical triangles $\triangle\mathrm{P'TC}$ and $\triangle\mathrm{LGC}$,[7] we have

$$\frac{\sin\widehat{\mathrm{P'T}}}{\sin\widehat{\mathrm{LG}}} = \frac{\sin\widehat{\mathrm{P'C}}}{\sin\widehat{\mathrm{LC}}} \Rightarrow \frac{\sin(90^\circ - \epsilon)}{\sin\left[90^\circ - \delta_1(90^\circ - \lambda)\right]} = \frac{\sin\left[90^\circ + \delta_2(\lambda)\right]}{\sin 90^\circ}.$$

Hence,

$$\frac{\cos\epsilon}{\cos\delta_1(90^\circ - \lambda)} = \frac{\cos\delta_2(\lambda)}{1} \Rightarrow \cos\delta_1(90^\circ - \lambda) = \frac{\cos\epsilon}{\cos\delta_2(\lambda)}.$$

Now, from equation (9), we can express $\sin\delta_1(90^\circ \pm \lambda)$ as $\sin\epsilon \cdot \sin(90^\circ \pm \lambda)$, or effectively, $\sin\epsilon \cdot \cos\lambda$. Hence,

$$\delta_1(90^\circ + \lambda) = \delta_1(90^\circ - \lambda), \tag{C2}$$

which makes, $\cos\delta_1(90^\circ \pm \lambda) = \dfrac{\cos\epsilon}{\cos\delta_2(\lambda)}$ agreeing with equation (C1).

6 These arcs, and their measures, are discussed in § 4.1.2.

7 See Van Brummelen (2013: 59–64) for the proof of the Rule of Four Quantities.

D ON FINDING THE TRUE DECLINATION IN MUNĪŚVARA'S *SIDDHĀNTASĀRVABHAUMA* I.4.41–42

MUNĪŚVARA, in his *Siddhāntasārvabhauma* (1646) 'The Emperor of all *siddhānta*s', discusses the computation of true declination of a celestial object in the section on the conjunction of planets with stars (*bhagrahayuti*). As Plofker (2002: 86) describes, Munīśvara claims the previously-stated methods of computing true declinations in *siddhāntic* texts are geometrically imprecise,[8] and hence provides the following alternative rule (in verse 41–42):[9]

ग्रहापमज्यात्रिगुणाभिघातस्त्रिभाढ्यखेटद्युगुणेन भक्तः ॥
फलस्य चापं ग्रहजोऽपमोऽन्योंऽशाद्यः स्वदिक्तच्छरयुग्वियोगौ ॥ ४१ ॥

एकान्यदित्त्वे भवतः क्रमेण तद्बाहुजीवात्रिभयुक्तखेटात् ॥
द्युजीवयाघ्नी त्रिगुणेन भक्ता तच्चापमंशाद्यपमः स्फुटः स्यात् ॥ ४२ ॥

grahāpamajyātriguṇābhighātas-
tribhāḍhyakheṭadyuguṇena bhaktaḥ ||
phalasya cāpaṃ grahajo 'pamo 'nyo
'ṃśādyaḥ svadik taccharayugviyogau || 41 ||

ekānyadiktve bhavataḥ krameṇa
tadbāhujīvātribhayuktakheṭāt ||
dyujīvayāghnī triguṇena bhaktā
taccāpam aṃśādyapamaḥ sphuṭaḥ syāt || 42 ||

The product of the Sine (*jyā*) of the [first] declination of the planet (*graha*) and the Radius (*triguṇa*) [i.e., the *sinus totus*] is divided by the day-Sine (*dyuguṇa*) [i.e., by the Cosine of the first declination] of [the longitude of] the planet (*kheṭa*) increased by three signs. The arc (*cāpa*) of the result (*phala*) is the other declination (*anya-apama*) [i.e., the second declination] connected with the planet (*graha*), beginning with degrees. The sum [or] difference [of the second declination] with the latitude (*śara*) in its own direction, 41...

...when they are in the same or different directions respectively, is [computed]. The Sine (*jīvā*) of its [appropriate] acute-angled arc (*bāhu*) multiplied by the day-Sine (*dyujīvā*) [i.e., by the Cosine of the first declination] of [the longitude of] the planet (*kheṭa*) increased by three signs is divided by the Radius; it's arc (*cāpa*), beginning with degrees, is the true declination (*sphuṭa-apama*). 42

8 Munīśvara's statement (from his auto-commentary the *Āśayaprakāśinī* or *Siddhāntatattvārtha*) denouncing the previously-stated methods as improper (*asaṃgata*) is discussed in § 1.2.2.

9 The numbering of the verses in the *bhagrahayutyadhikāra* of the *Siddhāntasārvabhauma* follows Ojhā (1978: 420).

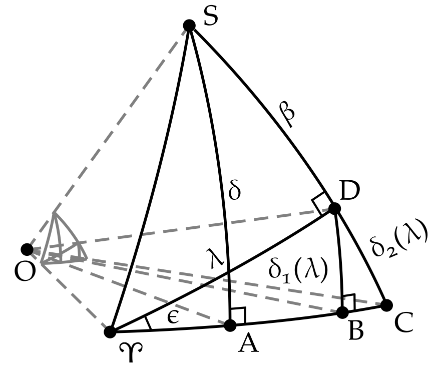

Figure D1: Spherical triangle $\triangle S♈D$ with its internal right spherical triangles: ∢♈AS, ∢SAC, ∢♈BD, ∢DBC, ∢♈DS, and ∢♈DC.

Munīśvara first describes the calculation of the other or second declination (*anya-apama*) in verse 41 and then goes on to state the method of computing the true declination (*sphuṭa-apama*) in verse 42. Figure D1 depicts a planet positioned at S in the celestial sphere with $\widehat{♈D}$ as it ecliptic longitude λ, $\widehat{SD}$ as it ecliptic latitude β, $\widehat{DB}$ as its first declination $\delta_1(\lambda)$, $\widehat{DC}$ as its second declination $\delta_2(\lambda)$, and ∢D♈C as the ecliptic obliquity or the measure of maximum declination ϵ. Applying the spherical law of sines to the spherical triangle △DBC, we find

$$\sin\delta_2(\lambda) = \frac{\sin\delta_1(\lambda)}{\sin\sphericalangle DCB} \Rightarrow \mathrm{Sin}\,\delta_2(\lambda) = \frac{\mathrm{Sin}\,\delta_1(\lambda)\cdot\mathcal{R}}{\mathrm{Cos}\,\delta_1(90^\circ+\lambda)},$$

since $\sphericalangle DCB = 90^\circ - \delta_1(90^\circ \pm \lambda)$, see equations (B2) and (C2).
Hence,

$$\widehat{DC} = \delta_2(\lambda) = \mathrm{arcSin}\left(\frac{\mathrm{Sin}\,\delta_1(\lambda)\cdot\mathcal{R}}{\mathrm{Cos}\,\delta_1(90^\circ+\lambda)}\right)_{\text{in degrees etc.}} \quad \text{(verse I.4.41).} \qquad \text{(D1)}$$

The value of $\delta_2(\lambda)$, taken in degrees etc., is added or subtracted to the latitude of the planet according to when they are in the same or different directions respectively (verse 41d–42a). In other words, $\delta_2(\lambda)+\beta$ when the second declination $\delta_2(\lambda)$ and the latitude β are both oriented in the same direction towards the north ecliptic pole or the south ecliptic pole, or alternatively, $\delta_2(\lambda)-\beta$ when the second declination $\delta_2(\lambda)$ and the latitude β are both oriented in different (opposing) directions.

Munīśvara does not name the result of the addition or subtraction of the latitude to the other/second declination; Nityānanda and Kamalākara, however, use the terms 'curve of true declination' (*sphuṭa-apama-aṅka*) and 'corrected second declination' (*spaṣṭa-anya-krānti*) respectively to refer to this quantity. Compare Nityānanda's and Kamalākara's discussions in § 4.2.1 and Appendix E respectively.

In verse 42b, Munīśvara clarifies how the measure $\delta_2(\lambda) \pm \beta$ is to be utilised in computing the true declination. First, an acute-angled arc called *bāhu* corresponding to this measure is calculated as follows:[10]

- *bāhu* is simply $\delta_2(\lambda) \pm \beta$ when the measure is less than 90°, and
- *bāhu* is $180° - \left[\delta_2(\lambda) \pm \beta\right]$ when the measure is greater than 90°.

With this appropriate acute-angled *bāhu*, Munīśvara then describes the true declination in verse 42cd as

$$\delta = \text{arcSin}\left(\frac{\left[\text{Sin}\,\delta_2(\lambda) \pm \beta\right] \cdot \text{Cos}\,\delta_1(90° + \lambda)}{\mathcal{R}}\right)_{\text{in degrees etc.}} \tag{D2}$$

Munīśvara's expression in equation (D2) is identical to Nityānanda's first method discussed in § 4.4, equation (30), and Kamalākara's procedure in his *Siddhāntatattvaviveka* VIII.23–24 discussed in Appendix E, equation (E3).

10 Nityānanda's statements on *bhuja* (*bāhu*) and *koṭi* are discussed in § 4.2.4.

E ON FINDING THE TRUE DECLINATION IN KAMALĀKARA'S *SIDDHĀNTATATTVAVIVEKA* VIII.21–25

IN HIS SIDDHĀNTATATTVAVIVEKA (1658) 'Investigation of the truth of *siddhānta*s', Kamalākara proposes a method to computing the true declination (*spaṣṭa-krānti*) of a celestial object in the section on the rising and setting (*udayāsta*) of celestial objects. Kamalākara first describes a method to calculate the second declination (*anya-krānti*, lit. other declination) and corrected second declination (*spaṣṭa-anya-krānti*, lit. corrected other declination) in verses 21–23ab, and using these quantities, calculates the true declination (*sphuṭa-krānti*) of the celestial object in verses 23cd-25.[11]

E.1 CALCULATING THE SECOND DECLINATION (*ANYA-KRĀNTI*) AND CORRECTED SECOND DECLINATION (*SPAṢṬA-ANYA-KRĀNTI*) IN VERSES 21–23AB

सत्रिभग्रहजद्युज्योद्धृता खेटापमज्यका ॥
त्रिज्यागुणाऽथ तच्चापमन्यक्रान्तिः स्वदिग्भवेत् ॥ २१ ॥

चलग्रहपरक्रान्तिज्ययोराहतिरुद्धृता ॥
सत्रिग्रहद्युमौर्व्या वा चापमन्यापमस्ततः ॥ २२ ॥

स्वेषुसंस्कारतः स्पष्टो भवेत्संस्कारदिक्क सः ॥ २३[प्र,द्वि]

satribhagrahajadyujyoddhṛtā kheṭāpamajyakā ||
trijyāguṇā 'tha taccāpam anyakrāntiḥ svadig bhavet || 21 ||

calagrahaparakrāntijyayor āhatir uddhṛtā ||
satrigrahadyumaurvyā vā cāpam anyāpamas tataḥ || 22 ||

sveṣusaṃskārataḥ spaṣṭo bhavet saṃskāradik ca saḥ || 23ab

The Sine (*jyā*) of the [first] declination (*apama*) of the planet (*kheṭa*) divided by the day-Sine (*dyujyā*) [i.e., by the Cosine of the first declination] produced of [the longitude of] the planet (*graha*) increased by three signs then multiplied by the Radius (*trijyā*) [i.e., by the *sinus totus*], the arc (*cāpa*) of that [value] is the other declination (*anya-krānti*) [i.e., the second declination] in its own direction. 21

Or, the product of the Sines (*jyā*) of the longitude of the planet (*cala-graha*, lit. moving planet') and the maximum declination (*para-krānti*) [i.e., the obliquity of the ecliptic] divided by the

11 The numbering of the verses in the *udayāstādhikāra* of the *Siddhāntatattvaviveka* follows K. C. Dvivedi (1993–8: pp. 172–174 in Part II).

day-Sine (*dyu-maurvī*) [i.e., by the Cosine of the first declination] of [the longitude of] the planet (*grahaja*) increased by three signs, the arc (*cāpa*) [obtained] from that [value] is the other declination (*anya-apama*). 22

By the correction (*saṃskāra*) of its own latitude (*iṣu*) [to the second declination], it becomes the true/corrected [other declination] (*spaṣṭa*-[*anya-krānti*]), indeed in the direction of the correction. 23ab

The verses 21–23ab describe two methods to compute the second declination $\delta_2(\lambda)$ of a planet. In verse 21, Kamalākara states

$$\delta_2(\lambda) = \text{arcSin}\left(\frac{\text{Sin}\,\delta_1(\lambda)\cdot\mathcal{R}}{\text{Cos}\,\delta_1(90^\circ+\lambda)}\right). \tag{E1}$$

This corresponds to $\overset{\frown}{DC}$ in Figure D1, and its derivation is discussed in the context of Munīśvara's method (from his *Siddhāntasārvabhauma* I.4.41) in equation (D1). It also appears in Nityānanda's *Sarvasiddhāntarāja* I.*tripraśnādhikāra*.49–50ab, see equation (B4).

To understand Kamalākara's second method, we can apply the spherical law of sines to right spherical triangle $\triangle\Upsilon DC$ in Figure D1:

$$\frac{\sin\overset{\frown}{DC}}{\sin\sphericalangle D\Upsilon C} = \frac{\sin\overset{\frown}{\Upsilon D}}{\sin\sphericalangle DC\Upsilon} \Rightarrow \sin\delta_2(\lambda) = \frac{\sin\lambda\cdot\sin\epsilon}{\sin\sphericalangle DC\Upsilon}.$$

Hence,

$$\text{Sin}\,\delta_2(\lambda) = \frac{\text{Sin}\,\lambda\cdot\text{Sin}\,\epsilon}{\text{Cos}\,\delta_1(90^\circ+\lambda)}. \tag{E2}$$

The arc Sine of the expression is the second measure of $\overset{\frown}{DC}$ or $\delta_2(\lambda)$ described in verse 22. The identification $\sphericalangle DC\Upsilon = \sphericalangle DCB = 90^\circ - \delta_1(90^\circ \pm \lambda)$ allows $\text{Cos}\left[\delta_1(90^\circ+\lambda)\right]$ to be the divisor in the equation above; see equations (B2) and (C2).

In the first hemistich of verse 23, Kamalākara describes how a correction of the latitude of the planet $\overset{\frown}{SD}$ or β added or subtracted to the second declination provides the value of the corrected second declination, i.e., $\overset{\frown}{SDC}$ or $\delta_2(\lambda)+\beta$ in Figure D1. More generally though, the corrected second declination can be expressed as $\delta_2(\lambda) \pm \beta$.[12]

12 Compare Nityānanda's discussion on the curve of true declination (*sphuṭa-apama-aṅka*) in § 4.2.1, and Munīśvara's statement on the appropriate choice of this arc-measure, i.e., the acute-angled arc called *bāhu*, on p. 159.

E.2 CALCULATING THE TRUE DECLINATION (*SPHUṬA-KRĀNTI*) IN VERSES 23CD–25

ग्रहकोटिद्युजीवाघ्नी तज्जीवा त्रिज्ययोद्धृता ॥ २३त्रि,च ॥

तच्चापं तु स्फुटाक्रान्तिः स्पष्टान्यापमदिक्स्थिता ॥
यद्वान्यापमजीवाप्ता स्फुटाऽन्यापमशिञ्जिनी ॥ २४ ॥

खेटापमज्यया निघ्नी चापं बिम्बस्फुटापमः ॥
तद्व्यत्ययात्स्फुटाख्यान्या क्रान्तिर्ज्ञेया बुधैरिह ॥ २५ ॥

grahakoṭidyujīvāghnī tajjīvā trijyayoddhṛtā || 23cd ||

taccāpaṃ tu sphuṭākrāntiḥ spaṣṭānyāpamadiksthitā ||
yadvānyāpamajīvāptā sphuṭānyāpamaśiñjinī || 24 ||

kheṭāpamajyayā nighnī cāpaṃ bimbasphuṭāpamaḥ ||
tad vyatyayāt sphuṭākhyānyā krāntir jñeyā budhair iha || 25 ||

The Sine (*jīvā*) of that [i.e., the Sine of true/corrected other declination] multiplied by the day-Sine (*dyujyā*) [i.e., by the Cosine of the first declination] of the complement (*koṭi*) of [the longitude of] the planet (*graha*) and divided by the Radius (*sinus totus*) 23cd...

...the arc (*cāpa*) of that [value] then is the true declination (*sphuṭa-krānti*), situated in the direction of the true/corrected other declination (*spaṣṭa-anya-apama*). Or, the Sine (*śiñjinī*) of the true/corrected other declination (*sphuṭa-anya-apama*) divided by the Sine (*jīvā*) of the other declination (*anya-apama*) 24...

...is multiplied by the Sine (*jyā*) of the [ecliptic first] declination (*apama*) of the planet; the arc (*cāpa*) [from that result] is the true declination (*sphuṭa-apama*) of the disk (*bimba*) [of the planet]. Contrary to this, the true/correct other declination (*sphuṭākhyā-anyā-krānti*) should be known [differently] by wise men in this case. 25

Here, Kamalākara proposes two methods to compute the true declination (*sphuṭa-apama*) of a celestial object. Verses 23cd–24ab suggest

$$\delta = \text{arcSin}\left(\frac{\text{Sin}\left[\delta_2(\lambda) \pm \beta\right] \cdot \text{Cos}\,\delta_1(90^\circ - \lambda)}{\mathcal{R}}\right). \tag{E3}$$

where delta is the arc $\widehat{SA}$ (in Figure D1, for the case of $\delta_2(\lambda) + \beta$). This method is identical to Nityānanda's first method described in § 4.5, equation (30) and

Munīśvara's method from his *Siddhāntasārvabhauma* I.4.42 in Appendix D, equation (D2). The identification $\text{Cos}\,\delta_1(90° - \lambda) = \text{Cos}\,\delta_1(90° + \lambda)$ follows from equation (C2) where the first declination of the complement of the longitude and the first declination of the longitude increased by ninety degrees are shown to be equal.

Verses 24cd–25ab present Kamalākara's second method to compute the true declination. Applying the spherical law of sines to the right spherical triangle $\triangle$DBC in Figure D1 gives

$$\frac{\sin \sphericalangle DCB}{\sin \widehat{DB}} = \frac{\sin \sphericalangle DBC}{\sin \widehat{DC}} \Rightarrow \sin \sphericalangle DCB = \frac{\sin \delta_1(\lambda)}{\sin \delta_2(\lambda)}. \tag{E4}$$

With $\sphericalangle DCB = \sphericalangle SCA$, the spherical law of sines applied to the right spherical triangle $\triangle$SCA (and using equation E4) yields

$$\frac{\sin \widehat{SA}}{\sin \sphericalangle SCA} = \frac{\sin \widehat{SC}}{\sin \sphericalangle SAC} \Rightarrow \sin \delta = \frac{\sin\left[\delta_2(\lambda) + \beta\right] \cdot \sin \delta_1(\lambda)}{\sin \delta_2(\lambda)}. \tag{E5}$$

Thus, by generalising the corrected second declination to $\delta_2(\lambda) \pm \beta$, and using a non-unitary Radius (*sinus totus*), we have

$$\text{Sin}\,\delta = \frac{\text{Sin}\left[\delta_2(\lambda) \pm \beta\right] \cdot \text{Sin}\,\delta_1(\lambda)}{\text{Sin}\,\delta_2(\lambda)}. \tag{E6}$$

The arc Sine of the expression is the second expression for the true declination of a celestial object. As Plofker (2002: 87) suspects, this method appears to be unique to Kamalākara's text. I have not found this exact expression attested in any Arabic, Persian, or Sanskrit works known to me.

REMARK

Both of Kamalākara's methods rely on the value of the second declination, corrected further by adding or subtracting the latitude of the planet according to the orientation of its orbit in the celestial sphere, to compute the true declination. The common factor in the right hand side of equations (E3) and (E6)—as well as equations (30) and (33) in Nityānanda's first two methods—is $\text{Sin}\left[\delta_2(\lambda) \pm \beta\right]$. To see the equivalence of the remaining factors in the right hand sides of equations (E3) and (E6),[13] we can restate equation (E2) as

$$\text{Sin}\,\delta_2(\lambda) = \frac{\text{Sin}\,\lambda \cdot \text{Sin}\,\epsilon}{\text{Cos}\,\delta_1(90° + \lambda)} = \frac{\text{Sin}\,\delta_1(\lambda) \cdot \mathcal{R}}{\text{Cos}\,\delta_1(90° + \lambda)} \quad \because \text{Sin}\,\delta_1(\lambda) = \frac{\text{Sin}\,(\lambda) \cdot \text{Sin}\,(\epsilon)}{\mathcal{R}}.$$

13 Appendix C discusses the equivalence between equations (30) and (33) in Nityānanda's first two methods.

In other words,

$$\frac{\text{Sin}\,\delta_1(\lambda)}{\text{Sin}\,\delta_2(\lambda)} = \frac{\text{Cos}\,\delta_1(90^\circ - \lambda)}{\mathcal{R}} \text{ or equation (E3)} \equiv \text{equation (E6)},$$

with $\delta_1(90^\circ - \lambda) = \delta_1(90^\circ + \lambda)$ from equation (C2).

GLOSSARY

This glossary lists Sanskrit technical expressions from the Sanskrit text of *Sarvasiddhāntarāja*, I.*spa·krā*. Individual entries are grouped together under their common English translation. At the end of each entry, appropriate verse-numbers indicate its location in § 3. The format of the glossary is described in § 2.3.

arc धनुस् (*dhanus*) 5, 6, 9; कोदण्ड (*kodaṇḍa*) 7; चाप (*cāpa*) 12

celestial equator विषुव-वृत्त (*viṣuva-vṛtta*) 5

celestial hemisphere गोल (*gola*) 10

celestial object नभोग (*nabhoga*) 4, 7; द्युचर (*dyucara*) 5; भ (*bha*) 7

circle वृत्त (*vṛtta*) 4

circle of asterisms भ-वलय (*bha-valaya*) 13

circle passing through the equinoctial points and the celestial object—

↪ circle congruent to the ecliptic भचक्र-सदृश-वृत्त (*bhacakra-sadṛśa-vṛtta*) 4; भवृत्त-सदृक्ष-वृत्त (*bhavṛtta-sadṛkṣa-vṛtta*) 5; भचक्र-सदृक्ष-वृत्त (*bhacakra-sadṛkṣa-vṛtta*) 6; भवृत्त-सदृश-वृत्त (*bhavṛtta-sadṛśa-vṛtta*) 7

congruent arc सदृश्-भुजा (*sadṛś-bhujā*) 7; सदृश-बाहु (*sadṛśa-bāhu*) 9

congruent complementary arc सदृश्-कोटि (*sadṛś-koṭi*) 7

conjunction of the equinoctial point and the node of the orbit of a celestial object विषुव-पात-युग (*viṣuva-pāta-yuga*) 6

Cosine of its latitude स्व-बाण-कोटिजीवा (*sva-bāṇa-koṭijīvā*) 8

Cosine of the first declination of the 'longitude increased by ninety degrees'—

↪ day-Sine [of the longitude] increased by three zodiacal signs स-भ-त्रय-द्युजीवा (*sa-bha-traya-dyujīvā*) 2

Cosine of the first declination of the longitude—

↪ day-Sine द्यु-जीवा (*dyu-jīvā*) 13

Cosine of the greatest declination परम-क्रान्ति-कोटिज्या (*parama-krānti-koṭijyā*) 3

Cosine of the second declination—

↪ Cosine of the other declination अन्य-क्रान्ति-कोटिज्या (*anya-krānti-koṭijyā*) 3

curve of true declination स्फुट-अपम-अङ्क (*sphuṭa-apama-aṅka*) 1 difference—

↪ difference अन्तर (*antara*) 1; विवर (*vivara*) 5, 6; वियुति (*viyuti*) 10

↪ made to be subtracted विशोधित (*viśodhita*) 11

direction दिश् (*diś*) 12

direction of the sum or the difference—

↪ direction of the conjunction or the disjunction युति-वियोग-दिश् (*yuti-viyoga-diś*) 11; संयोग-वियोग-दिश् (*saṃyoga-viyoga-diś*) 12

↪ own direction स्व-दिश् (*sva-diś*) 1

↪ same or different directions सम-भिन्न-दिश् (*sama-bhinna-diś*) 10

division

having been divided उद्धृत (*uddhṛta*) 2, 8

having been divided आप्त (*āpta*) 3

having been divided भाजित (*bhājita*) 9

ecliptic भवन-चक्र (*bhavana-cakra*) 6

ecliptic poles—

↪ ecliptic pole कदम्ब (*kadamba*) 13

↪ pair of ecliptic poles कदम्ब-युगल (*kadamba-yugala*) 4

equinoctial point विषुवत् (*viṣuvat*) 7

greater अधिक (*adhika*) 11

having been lowered अधरी-कृत (*adharī-kṛta*) 12

equivalent to त्रिभज्यका-उद्धृता (*tribhajyakā-uddh·rtaa*) *'having been divided by the Radius (60)'*

having been reduced from ninety नवतितश्च्युत (*navatitaś-cyuta*) 9

imprecise स्थूल (*sthūla*) 13

latitude—

↪ latitude बाण (*bāṇa*) 10, 13

↪ latitude of a celestial object खगस्य बाण (*khagasya bāṇa*) 1

lowered Sine of the congruent arc अधर-सदृक्ष-दोर्-ज्या (*adhara-sadrkṣa-dor-jyā*) 9 maximum latitude पर-इषु (*para-iṣu*) 6, 10; पर-शर (*para-śara*) 10

maximum true declination पर-स्फुट-अपम (*para-sphuṭa-apama*) 5

maximum true declination of a celestial object
ग्रहस्य पर-स्फुट-अपम (*grahasya para-sphuṭa-apama*) 11

multiplication
having been multiplied हत (*hata*) 2, 3, 8
multiplied गुण (*guṇa*) 12

ninety अभ्र-नव (*abhra-nava*)[14] 11; नवति (*navati*) 13

obliquity of the ecliptic—
↪ greatest declination परम-अपम (*parama-apama*) 10; परम-क्रान्ति (*parama-krānti*) 13

one direction एक-दिश् (*eka-diś*) 1

one hundred and eighty ख-अष्ट-भू (*akha-aṣṭa-bhū*)[15] 11

other declination अन्यतम-अपम (*anyatama-apama*) 1
synonymous with अन्य-क्रान्ति (*anya-krānti*) and identified with the द्वितीय-क्रान्ति (*dvitīya-krānti*) 'second declination'

pair of celestial poles ध्रुव-द्वय (*dhruva-dvaya*) 4

pair of equinoctial points विषुवत्-द्वय (*viṣuvat-dvaya*) 4

Radius त्रिभज्यका (*tribhajyakā*) 2, 8
synonymous with the *sinus totus*, and taken as 60 (in the *Sarvasiddhāntarāja*)

same सम (*sama*) 12

scholars of spherics—
↪ knower of spheres गोल-विद् (*gola-vid*) 4
↪ wise men who know the science of spheres गोल-विदुष (*gola-viduṣa*) 13

Sine of the complement of the arc of longitude of a celestial object
खगस्य कोटि-सिञ्जिनी (*khagasya koṭi-siñjini*) 8

Sine of the congruent arc सदृक्ष-बाहु-ज्यका (*sadṛkṣa-bāhu-jyakā*) 12

14 *bhūtasaṃkhyā* word-numerals, where *abhra* '0' and *nava* '9' make '90'.

15 *bhūtasaṃkhyā* word-numerals where *kha* '0', *aṣṭa* '8', and *bhū* '1' make '180'.

Sine of the congruent complementary arc सदृक्ष-कोटि-सिञ्जिनी (*sadṛkṣa-koti-siñjinī*) 8

Sine of the curve of true declination स्फुट-अपम-अङ्क-सिञ्जिनी (*sphuṭa-apama-aṅka-siñjinī*) 2; स्फुट-क्रान्ति-अङ्क-जिवा (*sphuṭa-krānti-aṅka-jīvā*) 3

Sine of the latitude of a celestial object नभोग-विशिखस्य सिञ्जिनी (*nabhoga-viśikhasya siñjinii*) 9

Sine of the maximum true declination पर-स्फुट-क्रान्ति-भव-ज्यका (*para-sphuṭa-krānti-bhava-jyakā*) 12

Sine of the true declination स्फुट-अपम-ज्यका (*sphuṭa-apama-jyakā*) 2; स्पष्ट-अपम-ज्यका (*spaṣṭa-apama-jyakā*) 3

solstitial colure—

↪ circle passing through the four poles ध्रुव-चतुष्क-यात-वृत्त (*dhruva-catuṣka-yāta-vṛtta*) 4

↪ solstitial colure आयन-वृत्त (*āyana-vṛtta*) 4, 5, 6, 7

sphere गोल (*gola*) 13

sum संयुति (*saṃyuti*) 1, 10

true declination स्फुट-अपम (*sphuṭa-apama*) 12; स्फुट-अपक्रम (*sphuṭa-apakrama*) 13; स्पष्ट-क्रान्ति (*spaṣṭa-krānti*) 13

well rounded सु-वृत्त (*su-vṛtta*) 4

Please write to ⟨wujastyk@ualberta.ca⟩ to file bugs/problem reports, feature requests and to get involved.

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