

Critical Remarks on Reference-Scaled Average Bioequivalence

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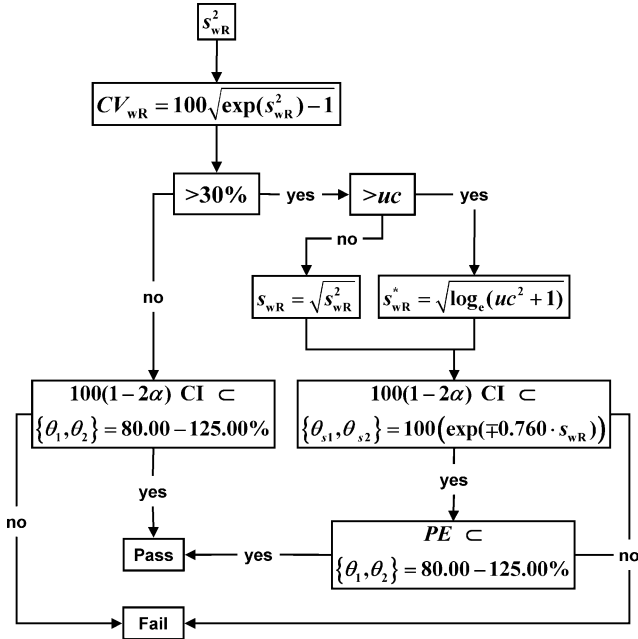


Fig. S1. Framework of Average Bioequivalence with Expanding Limits.

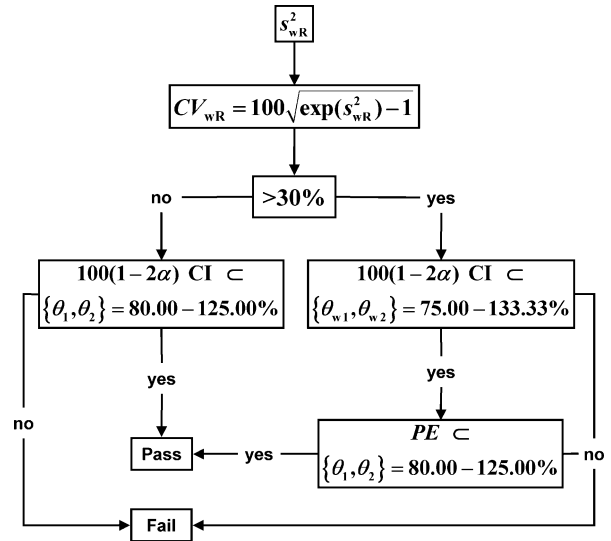


Fig. S2. Framework of the Gulf Cooperation Council.

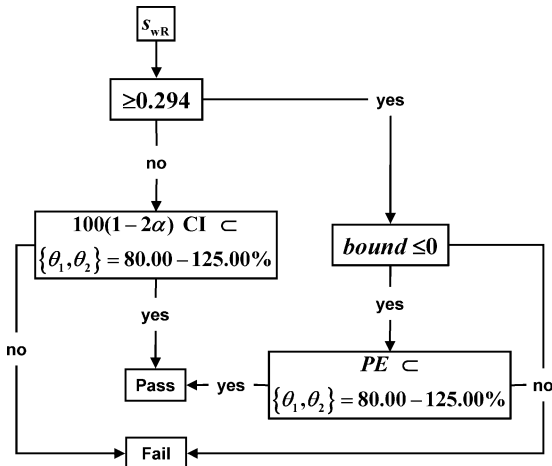


Fig. S3. Framework of Reference-Scaled Average Bioequivalence.

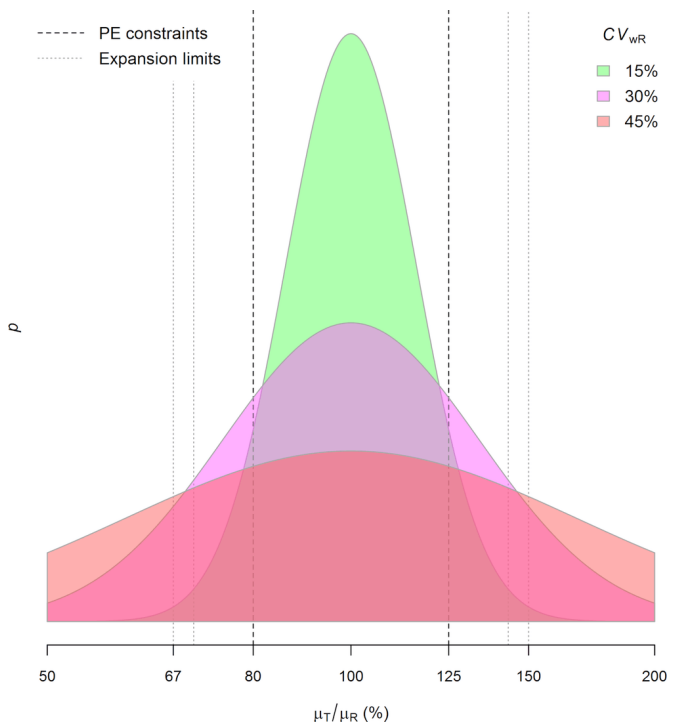


Fig. S4. Larger deviations between the geometric mean ratios are a direct consequence of higher variability. The point estimate constraint – together with the upper cap in most jurisdictions – leads to a truncated distribution whose statistical properties are essentially unknown. Hence, the test assuming the normal distribution of log_e-transformed data is not correct in the strict sense.

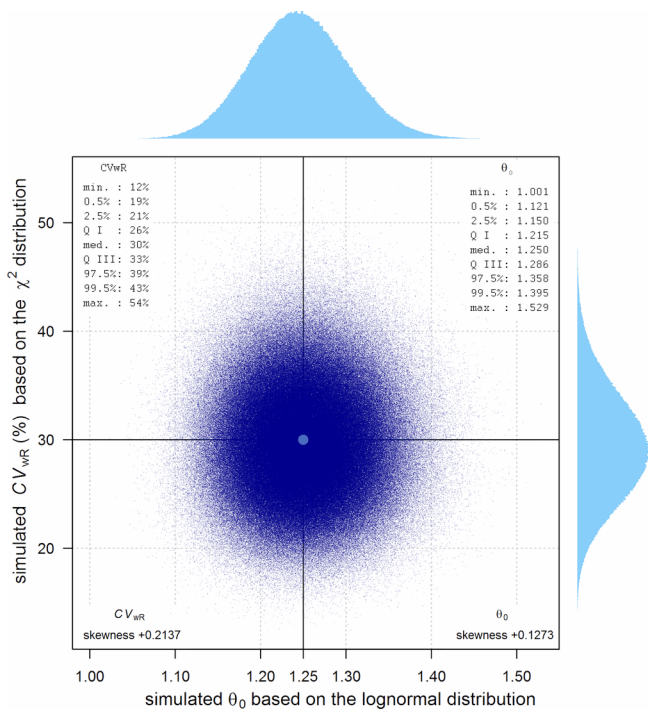


Fig. S5. Illustrative example of skewed distributions. One million simulations with true $CV_{wR} = 30\%$ and $\theta_0 = 1.25$ ($n = 24$).

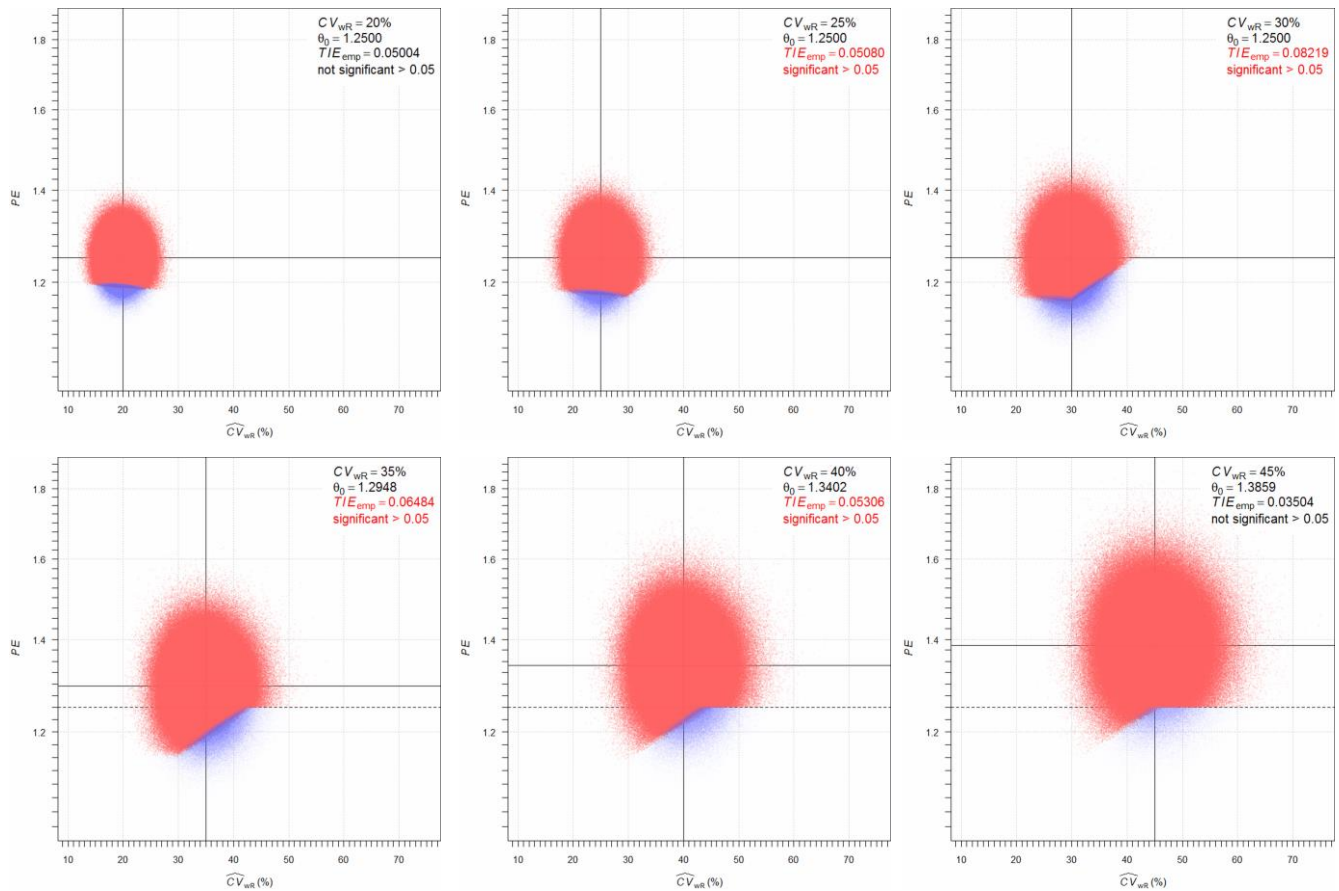


Fig. S6. One million simulations of studies in a two sequence, four period full replicate design ($n = 48$), assessment for ABEL ($uc = 50\%$). True $CV_{wR} = 20 - 45\% \rightarrow \theta_0 = \theta_{0s2}$ (solid lines). If applicable, upper PE -constraint 1.25 (dashed line). Y-axis in log-scale. Red dots represent studies which failed BE (null hypothesis correctly accepted), blue dots represent studies which passed BE (null hypothesis falsely rejected). Empiric Type I Error = number of passing studies / 10^6 , significance limit of the binomial test 0.050360.

R-script to reproduce Table 3

```

1 #####
2 # Parametric 95% confidence interval of CVWR based on #
3 # the chi2-distribution of its associated variance. #
4 #####
5 library(PowerTOST) # at least v1.1-0 (2013-02-08)
6 n <- seq(24L, 48L, 12L)
7 CVWR <- seq(20L, 65L, 5L)
8 txt <- "95% confidence intervals of CVWR\ndependent on the sample size.\n"
9 tmp <- data.frame(n = rep(n, each = length(CVWR)), CVWR = CVWR,
10                 lo = NA_real_, hi = NA_real_)
11 for (i in 1:nrow(tmp)) {
12   tmp[i, 3:4] <- round(100*CVCL(CV = tmp$CVWR[i] / 100, df = tmp$n[i] - 2,
13                               alpha = 0.05, side = "2-sided"), 1)
14 }
15 tmp <- reshape(tmp, direction = "wide", idvar = "CVWR", timevar = "n")
16 res <- data.frame(CVWR = CVWR, n.24 = NA_real_, n.36 = NA_real_, n.48 = NA_real_)
17 for (i in seq_along(CV)) {
18   res$n.24[i] <- sprintf("%5.1f \u2013 %5.1f", tmp[i, 2], tmp[i, 3])
19   res$n.36[i] <- sprintf("%5.1f \u2013 %5.1f", tmp[i, 4], tmp[i, 5])
20   res$n.48[i] <- sprintf("%5.1f \u2013 %5.1f", tmp[i, 6], tmp[i, 7])
21 }
22 names(res)[2:4] <- paste0(" n = ", n)
23 cat(txt); print(res, right = FALSE, row.names = FALSE)

```

Should give:

```

95% confidence intervals of CVWR
dependent on the sample size.
CVWR      n = 24      n = 36      n = 48
20    15.4 - 28.6  16.1 - 26.4  16.6 - 25.3
25    19.2 - 35.9  20.1 - 33.1  20.7 - 31.7
30    23.0 - 43.4  24.1 - 39.9  24.8 - 38.2
35    26.8 - 51.0  28.0 - 46.8  28.8 - 44.7
40    30.5 - 58.8  31.9 - 53.9  32.9 - 51.4
45    34.1 - 66.8  35.8 - 61.0  36.9 - 58.1
50    37.8 - 75.1  39.6 - 68.3  40.8 - 65.0
55    41.4 - 83.5  43.4 - 75.8  44.7 - 71.9
60    44.9 - 92.3  47.2 - 83.4  48.6 - 79.0
65    48.4 - 101.3 50.9 - 91.2  52.5 - 86.2

```

R-script to reproduce Table 4 (and sample sizes up to 144)

```

1 #####
2 # Empiric Type I Error at the #
3 # CVWR of maximum inflation. #
4 #####
5 library(PowerTOST) # At least v1.5-3 (2021-01-18) for GCC
6 design <- "2x2x4" # 2-sequence 4-period full replicate
7 n <- seq(24L, 144L, 12L) # Sample sizes
8 CV <- c(rep(0.30, 5 * length(n)), rep(se2CV(0.25), length(n))) # True CVWR
9 methods <- c("ABE", # ABE (all jurisdictions)
10             "ABEL1", # ABEL (EMA and others)
11             "ABEL2", # ABEL (Health Canada)
12             "ABEL3", # ABEL (GCC)
13             "RSABE1", # RSABE ('implied limits')
14             "RSABE2") # RSABE ('desired consumer risk model')
15 result <- data.frame(method = rep(methods, each = length(n)),
16                     reg = c(rep(NA, length(n)),
17                             rep("EMA", length(n)),
18                             rep("HC", length(n)),
19                             rep("GCC", length(n)),
20                             rep("FDA", 2 * length(n))),
21                     CV = CV, swr = CV2se(CV), n = n, U = 1.25,
22                     TIE = NA_real_, SE = NA_real_, signif = "no ")
23 limit <- binom.test(0.05 * 1e6L, 1e6L, alternative = "less")$conf.int[[2]]
24 pb <- txtProgressBar(0, 1, 0, width = NA, style = 3)
25 for (i in 1:nrow(result)) {
26   if (result$method[i] == "ABE") { # All jurisdictions (exact, no simulations)
27     result$TIE[i] <- power.TOST(CV = result$CV[i], n = result$n[i], design = design,
28                               theta0 = result$U[i])
29   }
30   if (result$method[i] == "ABEL1") { # EMA and others
31     result$U[i] <- scABEL(CV = result$CV[i], regulator = result$reg[i])[["upper"]]
32     result$TIE[i] <- power.scABEL(CV = result$CV[i], n = result$n[i], design = design,
33                                 theta0 = result$U[i], regulator = result$reg[i], nsims = 1e6L)
34   }
35   if (result$method[i] == "ABEL2") { # Health Canada
36     result$U[i] <- scABEL(CV = result$CV[i], regulator = result$reg[i])[["upper"]]
37     result$TIE[i] <- power.scABEL(CV = result$CV[i], n = result$n[i], design = design,
38                                 theta0 = result$U[i], regulator = result$reg[i], nsims = 1e6L)
39   }

```

```

40 if (result$method[i] == "ABEL3") { # Gulf Cooperation Council
41   result$U[i] <- scABEL(CV = result$CV[i], regulator = result$reg[i])[["upper"]]
42   result$TIE[i] <- power.scABEL(CV = result$CV[i], n = result$n[i], design = design,
43     theta0 = result$U[i], regulator = result$reg[i], nsims = 1e6L)
44 }
45 if (result$method[i] == "RSABE1") { # FDA 'implied limits'
46   result$U[i] <- scABEL(CV = result$CV[i], regulator = result$reg[i])[["upper"]]
47   result$TIE[i] <- power.RSABE(CV = result$CV[i], n = result$n[i], design = design,
48     theta0 = result$U[i], regulator = result$reg[i], nsims = 1e6L)
49 }
50 if (result$method[i] == "RSABE2") { # FDA 'desired consumer risk model'
51   if (result$swr[i] <= 0.25) {
52     result$U[i] <- scABEL(CV = result$CV[i], regulator = result$reg[i])[["upper"]]
53   } else {
54     result$U[i] <- exp(result$swr[i] * log(1.25) / 0.25)
55   }
56   result$TIE[i] <- power.RSABE(CV = result$CV[i], n = result$n[i], design = design,
57     theta0 = result$U[i], nsims = 1e6L)
58 }
59 setTxtProgressBar(pb, i / nrow(result))
60 }
61 result$signif[result$TIE > sign.limit] <- " yes "
62 result$SE <- sqrt(0.5 * result$TIE / 1e6)
63 result$SE <- sprintf("%.6f", result$SE)
64 ABE.idx <- which(result$method == "ABE")
65 ABEL1.idx <- which(result$method == "ABEL1")
66 ABEL2.idx <- which(result$method == "ABEL2")
67 ABEL3.idx <- which(result$method == "ABEL3")
68 RSABE1.idx <- which(result$method == "RSABE1")
69 RSABE2.idx <- which(result$method == "RSABE2")
70 result$CV <- sprintf("%.3f%", 100 * result$CV)
71 result$TIE <- sprintf("%.4f", result$TIE)
72 result$method[ABE.idx[1]] <- "ABE (all jurisdictions)"
73 result$method[ABE.idx[-1]] <- ""
74 result$method[ABEL1.idx[1]] <- "ABEL (EMA and others)"
75 result$method[ABEL1.idx[-1]] <- ""
76 result$method[ABEL2.idx[1]] <- "ABEL (Health Canada)"
77 result$method[ABEL2.idx[-1]] <- ""
78 result$method[ABEL3.idx[1]] <- "ABEL (GCC)"
79 result$method[ABEL3.idx[-1]] <- ""
80 result$method[RSABE1.idx[1]] <- "RSABE (\u2018implied limits\u2019)"
81 result$method[RSABE1.idx[-1]] <- ""
82 result$method[RSABE2.idx[1]] <- "RSABE (\u2018desired consumer risk model\u2019)"
83 result$method[RSABE2.idx[-1]] <- ""
84 result <- result[, -c(2, 4, 6)]
85 names(result)[2:6] <- c(" CVWR", " n", " TIE", " SE", "sign.")
86 print(result, row.names = FALSE, right = FALSE)

```

Should give:

method	CVWR	n	TIE	SE	sign.
ABE (all jurisdictions)	30.000%	24	0.0500	0.000158	no
	30.000%	36	0.0500	0.000158	no
	30.000%	48	0.0500	0.000158	no
	30.000%	60	0.0500	0.000158	no
	30.000%	72	0.0500	0.000158	no
	30.000%	84	0.0500	0.000158	no
	30.000%	96	0.0500	0.000158	no
	30.000%	108	0.0500	0.000158	no
	30.000%	120	0.0500	0.000158	no
	30.000%	132	0.0500	0.000158	no
	30.000%	144	0.0500	0.000158	no
ABEL (EMA and others)	30.000%	24	0.0804	0.000200	yes
	30.000%	36	0.0819	0.000202	yes
	30.000%	48	0.0823	0.000203	yes
	30.000%	60	0.0827	0.000203	yes
	30.000%	72	0.0831	0.000204	yes
	30.000%	84	0.0836	0.000204	yes
	30.000%	96	0.0834	0.000204	yes
	30.000%	108	0.0839	0.000205	yes
	30.000%	120	0.0838	0.000205	yes
	30.000%	132	0.0843	0.000205	yes
	30.000%	144	0.0840	0.000205	yes

ABEL (Health Canada)	30.000%	24	0.0841	0.000205	yes	
	30.000%	36	0.0846	0.000206	yes	
	30.000%	48	0.0846	0.000206	yes	
	30.000%	60	0.0852	0.000206	yes	
	30.000%	72	0.0851	0.000206	yes	
	30.000%	84	0.0855	0.000207	yes	
	30.000%	96	0.0855	0.000207	yes	
	30.000%	108	0.0855	0.000207	yes	
	30.000%	120	0.0855	0.000207	yes	
	30.000%	132	0.0857	0.000207	yes	
	30.000%	144	0.0857	0.000207	yes	
	ABEL (GCC)	30.000%	24	0.1493	0.000273	yes
		30.000%	36	0.1931	0.000311	yes
		30.000%	48	0.2324	0.000341	yes
30.000%		60	0.2621	0.000362	yes	
30.000%		72	0.2664	0.000365	yes	
30.000%		84	0.2668	0.000365	yes	
30.000%		96	0.2670	0.000365	yes	
30.000%		108	0.2684	0.000366	yes	
30.000%		120	0.2683	0.000366	yes	
30.000%		132	0.2689	0.000367	yes	
30.000%		144	0.2688	0.000367	yes	
RSABE ('implied limits')		30.000%	24	0.1335	0.000258	yes
		30.000%	36	0.1536	0.000277	yes
		30.000%	48	0.1708	0.000292	yes
	30.000%	60	0.1871	0.000306	yes	
	30.000%	72	0.2006	0.000317	yes	
	30.000%	84	0.2134	0.000327	yes	
	30.000%	96	0.2245	0.000335	yes	
	30.000%	108	0.2336	0.000342	yes	
	30.000%	120	0.2418	0.000348	yes	
	30.000%	132	0.2494	0.000353	yes	
	30.000%	144	0.2551	0.000357	yes	
	RSABE ('desired consumer risk model')	25.396%	24	0.0663	0.000182	yes
		25.396%	36	0.0629	0.000177	yes
		25.396%	48	0.0600	0.000173	yes
25.396%		60	0.0576	0.000170	yes	
25.396%		72	0.0557	0.000167	yes	
25.396%		84	0.0542	0.000165	yes	
25.396%		96	0.0532	0.000163	yes	
25.396%		108	0.0523	0.000162	yes	
25.396%		120	0.0517	0.000161	yes	
25.396%		132	0.0514	0.000160	yes	
25.396%		144	0.0508	0.000159	yes	

R-script to reproduce the example

```

1 #####
2 # AUC data from Reference (36) were evaluated in the R-package replicateBE v1.1.3 #
3 # (ABEL: WHO, GCC) and Phoenix winNonlin v8.3.4 (ABEL: HC, FDA: RSABE). #
4 #####
5 library(PowerTOST) # at least v1.5-3 (2021-01-18)
6 data <- data.frame(subject = c(rep(1L:2L, each = 4), rep(3L, 2), rep(4L:13L, each = 4),
7                               rep(15L:26L, each = 4), rep(27L, 2), rep(28L:40L, each = 4),
8                               rep(42L:50L, each = 4), rep(52L:57L, each = 4)),
9                               period = c(rep(1L:4L, 2), 1L:2L, rep(1L:4L, 10), rep(1L:4L, 12),
10                                          1L:2L, rep(1L:4L, 13), rep(1L:4L, 9), rep(1L:4L, 6)),
11                               sequence = c(rep("RTRT", 4), rep("TRTR", 4), rep("RTRT", 2), rep("TRTR", 4),
12                                             rep("RTRT", 8), rep("TRTR", 12), rep("RTRT", 4), rep("TRTR", 4),
13                                             rep("RTRT", 4), rep("TRTR", 4), rep("RTRT", 4), rep("TRTR", 4),
14                                             rep("RTRT", 8), rep("TRTR", 8), rep("RTRT", 4), rep("TRTR", 8),
15                                             rep("TRTR", 8), rep("TRTR", 6), rep("RTRT", 8), rep("TRTR", 4),
16                                             rep("RTRT", 4), rep("TRTR", 8), rep("RTRT", 8), rep("TRTR", 12),
17                                             rep("RTRT", 8), rep("TRTR", 8), rep("RTRT", 4), rep("TRTR", 4),
18                                             rep("RTRT", 4), rep("TRTR", 4), rep("RTRT", 12), rep("TRTR", 4),
19                                             rep("RTRT", 8), rep("TRTR", 8), rep("RTRT", 4)),
20                               treatment = NA_character_,
21                               AUC = c(812.6, 1173.7, 889.1, 620.1, 216.3, 338.0, 502.8, 398.6, 545.1, 542.9,
22                                       632.6, 520.0, 716.7, 860.4, 400.0, 223.8, 173.7, 289.7, 102.1, 185.3,
23                                       42.0, 88.3, 596.0, 659.3, 543.8, 662.9, 402.4, 359.8, 590.8, 444.3,
24                                       456.7, 378.4, 477.5, 407.9, 304.5, 351.5, 520.2, 335.7, 500.7, 323.0,
25                                       416.3, 525.1, 176.1, 710.7, 409.5, 645.5, 160.6, 218.0, 170.1, 124.6,
26                                       562.4, 490.4, 504.7, 675.9, 756.0, 606.8, 477.4, 626.8, 207.5, 271.6,
27                                       173.7, 240.5, 571.3, 705.2, 619.0, 633.6, 511.9, 549.7, 388.2, 141.0,
28                                       124.0, 91.9, 113.3, 59.5, 536.1, 595.2, 445.5, 521.5, 239.7, 265.1,
29                                       445.9, 433.2, 609.6, 371.6, 511.3, 432.7, 449.9, 860.4, 606.8, 577.2,
30                                       192.5, 220.1, 233.1, 227.0, 764.4, 508.8, 757.8, 449.4, 151.9, 194.8,
31                                       568.1, 321.1, 338.3, 403.6, 735.6, 634.5, 1244.2, 641.9, 429.1, 391.8,
32                                       316.9, 335.1, 307.4, 481.8, 346.6, 369.7, 409.0, 514.6, 763.1, 406.5,
33                                       271.0, 221.0, 296.5, 463.7, 292.9, 431.0, 448.5, 267.8, 217.2, 332.2,
34                                       103.0, 127.5, 290.8, 208.6, 243.7, 489.7, 297.2, 502.0, 320.4, 334.3,

```

```

35      163.8, 232.1, 636.9, 434.9, 368.3, 292.6, 446.1, 222.3, 193.7, 202.8,
36      255.2, 244.3, 534.1, 243.1, 418.4, 441.9, 355.1, 415.2, 382.7, 334.0,
37      102.0, 282.5, 245.6, 286.2, 320.5, 233.9, 331.7, 260.5, 223.6, 645.4,
38      349.0, 507.4, 504.5, 289.9, 550.7, 244.2, 615.8, 732.1, 620.9, 665.2,
39      898.4, 924.9, 398.3, 828.3, 410.4, 329.2, 449.4, 442.1, 237.0, 505.0,
40      496.3, 580.6, 332.4, 273.6, 525.3, 293.3, 185.2, 222.9, 182.1, 194.1,
41      246.9, 620.9, 678.3, 752.2, 235.4, 190.4, 318.3, 248.4, 180.6, 174.7,
42      102.9, 117.0))
43 for (i in 1:nrow(data)) { # extract treatments from sequences and periods
44   treatments <- unlist(strsplit(data$sequence[i], split = ""))
45   data$treatment[i] <- treatments[data$period[i]]
46 }
47 nsims <- 1e6L # number of simulations for empiric Type I Error
48 sign.limit <- binom.test(0.05 * nsims, n = nsims, alternative = "less")$conf.int[[2]]
49 txt <- paste0("Significance limit based on the binomial test for ",
50             formatc(1e6L, digits = 0, big.mark = ","), " simulations = ",
51             sprintf("%.6f", sign.limit), ".\n")
52 method <- c("ABEL (WHO)", # ANOVA (EMA 'Method A')
53            "ABEL (GCC)", # ANOVA
54            "ABEL (HC)", # mixed-effects model
55            "RSABE (FDA)") # ANDA guidance
56 regulator <- c("EMA", "GCC", "HC", "FDA")
57 result <- data.frame(method = method, regulator = regulator,
58                    CVWR = c(rep(0.355648, 3), 0.353968),
59                    L = NA_real_, U = NA_real_,
60                    lower = c(rep(1.017529, 2), 1.016937, NA_real_),
61                    upper = c(rep(1.174621, 2), 1.175704, NA_real_),
62                    PE = c(rep(1.093257, 2), 1.093257, 1.097784),
63                    Delta.r = NA_real_,
64                    bound = c(rep(NA_real_, 3), -0.0555212),
65                    BE = "fail", TIE = NA_real_, signif = "",
66                    stringsAsFactors = FALSE) # last line required only for R <4.0
67 for (i in seq_along(method)) {
68   if (method[i] == "ABEL (WHO)") {
69     result[i, 4:5] <- scABEL(CV = result$CVWR[i], regulator = regulator[i])
70     result$Delta.r[i] <- 1 - result$L[i]
71     if (result$lower[i] >= result$L[i] & result$upper[i] >= result$U[i] & # check CI inclusion
72         result$PE[i] >= 0.80 & result$PE[i] <= 1.25) # check PE constraint
73       result$BE <- "pass"
74     result$TIE[i] <- power.scABEL(CV = result$CVWR[i], n = 54, design = "2x2x4",
75                                theta0 = result$U[i], regulator = regulator[i], nsims = nsims)
76     if (result$TIE[i] > limit) result$signif[i] <- "*" # check significant TIE
77   }
78   if (method[i] == "ABEL (GCC)") {
79     result[i, 4:5] <- scABEL(CV = result$CVWR[i], regulator = regulator[i])
80     result$Delta.r[i] <- 1 - result$L[i]
81     if (result$lower[i] >= result$L[i] & result$upper[i] >= result$U[i] & # check CI inclusion
82         result$PE[i] >= 0.80 & result$PE[i] <= 1.25) # check PE constraint
83       result$BE <- "pass"
84     result$TIE[i] <- power.scABEL(CV = result$CVWR[i], n = 54, design = "2x2x4",
85                                theta0 = result$U[i], regulator = regulator[i], nsims = nsims)
86     if (result$TIE[i] > limit) result$signif[i] <- "*" # check significant TIE
87   }
88   if (method[i] == "ABEL (HC)") {
89     result[i, 4:5] <- scABEL(CV = result$CVWR[i], regulator = result$regulator[i])
90     result$Delta.r[i] <- 1 - result$L[i]
91     if (result$lower[i] >= result$L[i] & result$upper[i] >= result$U[i] & # check CI inclusion
92         result$PE[i] >= 0.80 & result$PE[i] <= 1.25) # check PE constraint
93       result$BE <- "pass"
94     result$TIE[i] <- power.scABEL(CV = result$CVWR[i], n = 54, design = "2x2x4",
95                                theta0 = result$U[i], regulator = regulator[i], nsims = nsims)
96     if (result$TIE[i] > limit) result$signif[i] <- "*" # check significant TIE
97   }
98   if (method[i] == "RSABE (FDA)") {
99     result[i, 4:5] <- scABEL(CV = result$CVWR[i], regulator = regulator[i])
100    result$Delta.r[i] <- 1 - result$L[i]
101    if (result$bound[i] <= 0 & # check bound
102        result$PE[i] >= 0.80 & result$PE[i] <= 1.25) # check PE constraint
103      result$BE <- "pass"
104    result$TIE[i] <- power.RSABE(CV = result$CVWR[i], n = 54, design = "2x2x4",
105                               theta0 = result$U[i], regulator = regulator[i], nsims = nsims)
106    if (result$TIE[i] > limit) result$signif[i] <- "*" # check significant TIE
107  }
108 }
109 result <- result[, -which(names(result) == "regulator")]
110 result$CVWR <- sprintf("%.2f%%", 100 * result$CVWR)
111 result$L <- sprintf("%.2f%%", 100 * result$L)
112 result$U <- sprintf("%.2f%%", 100 * result$U)
113 result$lower <- sprintf("%.2f%%", 100 * result$lower)
114 result$upper <- sprintf("%.2f%%", 100 * result$upper)
115 result$PE <- sprintf("%.2f%%", 100 * result$PE)
116 result$Delta.r <- sprintf("%.2f%%", 100 * result$Delta.r)
117 result$bound <- signif(result$bound, 4)
118 result$TIE <- signif(result$TIE, 3)
119 result[1:3, 9] <- "\u2013"
120 result[4, 5:6] <- "\u2013"

```

121 `cat(txt); print(result, row.names = FALSE)`

Should give:

Significance limit based on the binomial test for 1,000,000 simulations = 0.050360.

	method	CVwR	L	U	lower	upper	PE	Delta.r	bound	BE	TIE	signif
ABEL	(WHO)	35.56%	76.93%	129.99%	101.75%	117.46%	109.33%	23.07%	-	pass	0.0643	*
ABEL	(GCC)	35.56%	75.00%	133.33%	101.75%	117.46%	109.33%	25.00%	-	pass	0.0459	
ABEL	(HC)	35.56%	76.93%	129.99%	101.69%	117.57%	109.33%	23.07%	-	pass	0.0651	*
RSABE	(FDA)	35.40%	73.59%	135.89%	-	-	109.78%	26.41%	-0.05552	pass	0.0232	